International Electronic Journal of Elementary Education, 2015, 7(3), 301-318.

A Study of Geometry Content Knowledge of Elementary Preservice Teachers

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Received: 3 March 2015 / Revised: 17 May 2015 / Accepted: 27 May 2015

Abstract

The purpose of this research is to examine preservice elementary school teachers' geometry learning as investigated by both qualitative and quantitative methods. For the qualitative investigation, narrative analysis and thematic analysis methods were used. The findings of narrative analysis indicated two main kinds of stories: as a learner and as a beginning teacher. The thematic analysis findings yield to three themes: history of learning geometry, perceptions about geometry, effective geometry instructional practices. The findings informed the quantitative investigation on geometry content knowledge for the case of quadrilaterals. During the second phase of the study, 102 participants who enrolled in the methods course completed pre and post test of teachers' geometry content knowledge. Treatment group participants (n=54) received series of activities (geometry activities and student work analysis) focusing on quadrilaterals, and control group participants (n=48) received traditional instruction. Repeated measures ANOVA results showed a significant change in treatment group participants' geometry content knowledge. The mixed ANOVA results indicated a significant main effect of knowledge but no significant interaction between geometry content knowledge and grouping. Even though treatment group participants' geometry content knowledge growth was significant, the difference between treatment group and control group participants' growth in geometry content knowledge was not significant. This study informs mathematics teacher education in three important areas; limited knowledge of preservice teachers' geometry content knowledge, integrating mathematics content and the context of teaching into methods course, and use of student work with preservice teachers.

Keywords: Teachers' mathematics content knowledge, geometry, mathematical knowledge for teaching, elementary school preservice teachers.

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ISSN:1307-9298

Christiana was excited to go to her first class in university after transferring from the community college of the same city. She was hopeful to be a good teacher. On her way to mathematics course, she remembered her mathematics teachers throughout her education. She regretted that none of them had inspired her to learn mathematics. She wanted to have a new start with this university because she cared about her future students from then. She wanted to learn mathematics that she previously avoided, and she wanted to know everything about teaching mathematics to be the good teacher that she never had.

Christiana is one of the participants who told her story of learning geometry for the study discussed in this manuscript. This article reports a two-phase research study which integrated qualitative and quantitative research methods to study preservice elementary teachers' geometry learning and their geometry content knowledge. The first phase of the study was the qualitative investigation to understand preservice teachers' geometry learning. Integration of results from the study of effective geometry learning experiences of preservice teachers and teacher education literature, the researcher developed series of activities for a mathematics methods course. Those activities used as the intervention for the quasi-experimental quantitative phase with purpose of improving the geometry content knowledge of preservice teachers. This article will report (i) the qualitative investigation on preservice elementary teachers' geometry learning, (ii) the development of the activities as a result of that investigation, and (iii) studying the effect of the activities by a quantitative investigation.

Introduction

The most commonly accepted definition of teacher knowledge was given by Shulman (1986, 1987), who developed a model of teacher knowledge. His definition is consisted of three types of teacher knowledge: subject matter knowledge (SMK), pedagogical content knowledge (PCK) and curriculum knowledge. SMK refers to knowledge base of the content one is teaching, such as mathematics. PCK "goes beyond knowledge of subject matter per se to the dimensions of subject matter knowledge for teaching" (Shulman, 1986, p. 9). PCK is the type of knowledge that distinguishes the work of a teacher from the work of a scientist. The third knowledge type, curriculum knowledge, addresses effective use of curriculum materials and teachers' familiarity with other subjects studied.

Among these knowledge types, subject matter knowledge stands out as a point of interest for teacher education. Brown and Borko (1992) asserted that preservice teachers' limited mathematics content knowledge may hinder their pedagogical training. Also, other studies have shown that lack of subject matter knowledge affects teacher's methods of teaching (e.g. Carpenter, Fennema, Peterson & Carey, 1988; Leinhardt & Smith, 1985). Carpenter and his colleagues (1988) emphasized that subject matter knowledge of a teacher heavily affects the teachers' use of the pedagogical tools. Even though SMK is emphasized greatly in teacher knowledge, the type of mathematics is not just to solve problems mathematically correct (Ball, 1988, 1990a, 1990b; Leinhardt and Smith, 1985; Owens, 1987; Post, Harel, Behr, & Lesh, 1988; Steinberg, Haymore, & Marks, 1985).

In the mathematics education field, Ball and a group of researchers developed mathematical knowledge for teaching (MKT) as following the Shulman's model for teacher knowledge. MKT model addresses how a teacher uses mathematics for teaching while emphasizing the importance of mathematics knowledge in teaching settings (Ball, 2000). According to MKT model, there are six domains of teacher's content knowledge which can be categorized under Shulman's different types of knowledge (Ball, Thames & Phelps, 2008). There are three domains under subject matter knowledge: common content knowledge (CCK, mathematics knowledge not unique to teaching), specialized content knowledge (SCK, mathematics knowledge unique to teaching), and horizon content knowledge (knowledge of mathematics throughout the curriculum). Also, there are three domains under pedagogical content knowledge: knowledge of content and students (KCS, interaction of knowledge of mathematics and students' mathematical conceptions), knowledge of content and teaching (KCT, interaction of knowledge of mathematics and teaching methods), and knowledge of content and curriculum (interaction of knowledge of mathematics and mathematics curriculum). This model was used wide spread in mathematics education research. There were also efforts to adapt or improve the model according to different contexts. For the international comparison study on preservice mathematics teachers (Tatto et al., 2008), MKT model and the teacher knowledge instrument inspired TEDS-M study. Furthermore, Mathematics Teachers' Specialized Knowledge (MTSK) was developed by Carillo and his colleagues in order to strengthen the connection to classroom practices (Carrillo, Climent, Contreras, & Muñoz-Catalán, 2013).

Content knowledge of teachers is important for every subject including geometry, one of the most applicable topics to daily life, yet, which is often a neglected topic in the curriculum. There are several studies on teachers' knowledge of mathematics focused on topics such as fractions (Carpenter et. al, 1988) or numbers and operations (Ball, 1990; Ma, 1999). The limited number of research projects focused on knowledge of geometry for teaching concludes that beginning teachers are not equipped with necessary content and pedagogical content knowledge of geometry, and it is important to address this issue in teacher education (Browning, Edson, Kimani, Aslan-Tutak, 2014; Jones, 2000; Swafford, Jones, & Thornton, 1997).

Studies on geometry content knowledge of teachers emphasized the lack of teachers' knowledge, especially the beginning teachers (Barrantes & Blanco, 2006; Chinnappan, Nason, & Lawson, 1996; Jacobson & Lehrer, 2000; Lampert, 1988; Leikin, Berman, & Zaslavsky, 2000). "Teachers are expected to teach geometry when they are likely to have done little geometry themselves since they were in secondary school, and possible little even then." (Jones, 2000, p. 110).

Therefore, this study is an effort to improve mathematics teacher education in geometry. This study's most important characteristic is to understand preservice teachers' needs and strengths from their perspective in order to address their geometry learning needs to enhance their geometry content knowledge. There were four research questions, first two to address geometry learning (studied by qualitative research methods) and the last two to address geometry content knowledge of preservice teachers (studied by quantitative research methods):

1. What are participating preservice elementary teachers' perceptions of geometry in elementary school who were enrolled in mathematics methods course?
2. What are the perceptions of participating preservice elementary teachers on effective instructional strategies to promote their knowledge of geometry in the mathematics methods courses?
3. Does use of geometry activities focused on quadrilaterals with analysis of student work influence preservice elementary teachers' geometry content knowledge?
4. Is there a significant difference in geometry content knowledge between preservice teachers who received regular mathematics methods course instruction and preservice teachers who received experimental mathematics methods course instruction?

Method

This study was conducted in a mathematics methods course at a large southeastern research university for predominantly middle-class, white, female elementary school preservice teachers in the U.S. Students begin their unified elementary education program in their junior year and usually they take the methods course in their senior year. This course plays an important role in preservice teachers' education because it is the only mathematics methods course for elementary school preservice teachers.

Before taking the mathematics methods course, elementary school preservice teachers are required to take three mathematics courses, two general mathematics courses (e.g. calculus) and one content course for elementary teachers. The mathematics content course addresses mathematics concepts for elementary school level whereas the mathematics methods course is designed to build the future teachers' pedagogical tools for teaching mathematics. Even though the recommendation of this order is given, some students take methods course and the content course at the same time or some take methods course before the content course. The research reported in this manuscript is multi-stage, using mixed research methods. The research was conducted in three phases, Figure 1. Due to nature of research itself, the manuscript is also organized in three sections.

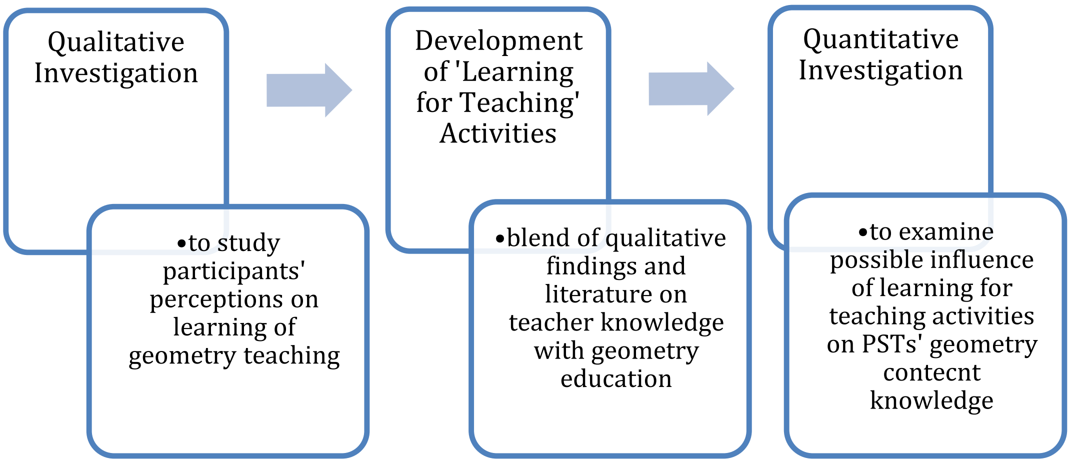

Figure 1. Three phases of the study.

Phase I: Qualitative investigation

The theoretical perspective of this investigation is constructivism. Hatch (2002) addressed the quest of a constructivist researcher as "individual constructions of reality compose the knowledge of interest to constructivist researcher" (p.15). For this research, in order to study preservice teachers' geometry knowledge, first, the researcher listened preservice teachers about their experiences of learning of geometry and their perspective on means to improve their geometry content knowledge. It was necessary to address preservice teachers' constructions of geometry learning in order to be able to develop tasks to improve their geometry content knowledge.

The goal of the qualitative investigation was to understand preservice teachers' geometry learning especially in methods courses. The findings of the first phase informed teacher education practice to develop geometry activities for methods course to be used in the third phase (quantitative investigation). Christiana, Emma and Liz (pseudonyms), the volunteered participants, were preservice elementary school teachers who were enrolled in the methods course. There was one participant from each three sections of the course. The researcher was not offering methods course at the time of qualitative investigation. In this study, only Liz took the content course before methods course. The other two participants, Christiana and Emma were planning to take it the following semester.

Qualitative data sources

The data collection methods included individual interviews with the participants, observations of geometry instruction in each section of the course for two weeks, and the collection of materials used during the geometry instructions. Field notes were taken during the observations. Also, copies of the instructional materials (handouts and transparencies) and student presentations were collected. The primary purpose of the observations and the artifact collection was to capture content preparation for the geometry learning process of preservice elementary teachers in order to provide triangulation for the interview data. The primary data source for this investigation was individual interviews. The purpose of the interviews was to study preservice elementary teachers' stories of learning geometry. The 45-60 minutes long interviews were conducted after the participants received geometry instruction in methods course.

The narrative interview protocol was used which was designed to be semi-structured and open-ended. The narrative interviews are tailored to intrigue story telling from participants through open-ended questions or probes (Reissman, 1993, 2000). The mostly suggested narrative interview probes are "Tell me about…" (Reissman, 1993, 2000). For this study, some of the interview questions were "Tell me about your geometry learning before college" or "Tell me about geometry instruction in methods course". Another important feature of narrative interviews is that the researcher accepts the leading role of the participant because the participant is the knowledge holder (Bruner, 1990; Reissman, 2000).

Qualitative data analysis

The data analysis in this qualitative investigation was focused on participants' experiences of geometry learning. The interviews, the source of the data analysis, were analyzed for both narrative and non-narrative forms. In addition to structural analysis of the preservice teachers' stories (Labov, 1972) thematic analysis (Coffey & Atkinson, 1996) of both narrative and non-narrative data was used.

Individuals may use narratives for meaning making in addition to using them for sharing their experiences in stories (McAdams, 1993; Reissman, 1993). Grbich (2007) identified research settings which might be addressed by narrative analysis as "those that explore either the structure of narratives or the specific experiences of particular events, e.g. marriage breakdown; finding out information which is life changing; undergoing social/medical procedures; or participating in particular programmes" (p. 124). In the case of teacher learning, narrative analysis may be used to study professional development experiences of in-service teachers or preservice teachers in teacher education programs. Also, Cortazzi (1993) suggests that teachers may prefer to discuss their learning and their knowledge through stories. Teachers' narratives have been used in teacher education and teacher development in various context such as Carter (1993), Clandinin and Connelly (1996), Cortazzi (1993), Doyle and Carter (2003), and Elbaz (1991). Sarac (2012) used semi-structured narrative interviews in order to categorize teachers in terms of their teaching self-efficacy levels. "Researchers have come to appreciate that teachers' stories offer a wealth of information about their individual identities and classroom experiences" (Lloyd, 2006, p. 58).

The stories told by participants during the interviews were analyzed by using narrative analysis method of Labov (1972). According to Labov (1972, 1982) a narrative has a structure and a sequence. If a narrative is fully formed, it has six components; abstract (AB; summary of the narrative), orientation (OR; time, place people etc.), complicating action (CA; sequence, turning points, crisis, content), resolution (RE; resolution of events, crisis), evaluation (EV; interpretation), and coda (CO; narrative ends and turn back to listener). The structure of the narratives, produced by participants, gives insights about how they perceive their experiences in methods course. The order of the components may change, while some of the components may be absent from stories. Table 1 provides an example of Labov's narrative coding on the story of a participant about her content course experiences.

Table 1. *An example of narrative coding*

this is really where it gets tricky	**AB**
I did not like the teacher (.)	
I don't think she (.) taught the class very well (.)	
she already had a notebook of notes	**OR**
you have for the rest of the year and	
she followed it very strictly and	
if you would ask a question	**CA**
she would just say either come and see me after class or	
she would like no its right there you are supposed to get it and	
she kept going on so our questions were unanswered and	**RE**
I really didn't like that and	**EV**
she just she just didn't have a lot of patience and	

*****AB**: abstract, **OR**: orientation, **CA**: complicating action, **RE**: resolution, **EV**: evaluation, **CO**: coda

In addition to structural analysis of narratives, thematic analysis (Coffey & Atkinson, 1996) was used and the whole interviews were coded. Literature supports using other analysis methods in addition to narrative analysis in order to deepen the analysis of the rich data (Lloyd, 2005, 2006; Reissman, 1993, Robichaux, 2002). In addition to the narratives, participants talked about geometry learning and teaching in non-narrative form. The open codes from interviews yielded into themes to inform the researchers about effective geometry learning experiences for the participants.

Qualitative findings

The findings section of the qualitative investigation is organized as narrative analysis findings and thematic analysis findings. There were two main kinds of stories with sub headings emerged from participants' narratives: stories as a learner and stories as a beginning teacher. The thematic analysis yielded three themes from preservice teachers' geometry learning: history of learning geometry, perceptions about geometry, effective geometry instruction approaches.

Narrative analysis findings

The participants told stories about their learning experiences of geometry from two different perspectives, as a learner (K-12 and college mathematics courses) and as a beginning teacher (college mathematics courses and mathematics methods course). Even though participants experienced the methods course as beginning teachers, all three of the participants emphasized the role of their history of learning geometry as a student on their experiences in the methods course as beginning teachers. Therefore, the stories from both perspectives (learner and beginning teacher) are important to study in order to understand preservice elementary mathematics teachers' geometry learning in mathematics methods course.

The resolution (RE) and evaluation (EV) components of the narratives reflected the focus of the participants as a learner or as a beginning teacher in addition to participants' perceptions about geometry learning. In addition to RE and EV components, the OR component informed the researcher about the settings, time and characteristics of the instructions in the narratives. One interesting result from orientation competent of narratives from all three participants was that all of the narratives were about courses that participants took. The participants did not tell any story outside the formal education environment, even though geometry has strong connection with real life applications.

Stories as a learner. The stories of learning geometry with an emphasis as a learner were stressed usually in K-12 education and in college mathematics courses. For example, Emma mentioned about the geometry course that she took in 9th grade and her perceptions about that class. "we did I remember making bridges and to see how much weight popsicles sticks with different shapes and angles how to build together stuff and I didn't love it (.) I didn't really take another I don't think we really did a lot of geometry".

On the other hand, for college mathematics courses participants told stories from both perspectives, as a learner and as a beginning teacher. All three participants told stories from the mathematics courses they took and they expressed that those courses were as a review of their high school knowledge. Only Christiana expressed that one of the college mathematics course was effective in her learning. Due to her weak mathematics background from high school and community college, she expressed that she learned more mathematics in that college mathematics course than in high school mathematics courses. "in topics of mathematics it went through everything it went through like statistics geometry algebra stuff that I never heard of truth tables".

The stories told about the mathematics content course for elementary school teachers is limited because only one participant, Liz, took the course before the methods course. The stories of Liz from that course reflected her concerns about the limited mathematics learning and through the absence of the connection of that course to her teaching career. Liz was concerned that she could not learn enough. Also, her story of geometry learning in that class expressed that the content was confusing for her. "we reviewed the properties of parallelograms what makes them rhombus and stuff a drawing of each of these things but she really lightly touched on them like on their characteristics she did not spend a lot of time on talking about distinctions so sometimes we would be confused wait so is this this (emphasized) or is this that (emphasized) she goes like that its that and just keep going and so its never stop I didn't get it".

In spite of her focus in methods course as a beginning teacher Liz expressed that her experiences as a student in the methods course was more effective than the content course for learning mathematics. "even if math was challenging she [methods course instructor] makes it so that get it and she would go back and explain it in other way…what I like this class a lot better than [the content course] I like concrete models and I like different ways of looking at the same thing".

Stories as a learner. Since the participants took their college mathematics courses after they decided to be teacher, they had the consciousness about learning mathematics in those courses as a teacher. The beginning teacher aspect, being able to relate college education into elementary classroom teaching, was briefly expressed in the narratives from mathematics courses. An example of the beginning teacher aspect is Liz's perspective on mathematics content course. Even though her priority in that course was to learn mathematics as a student, she had thoughts about ways to transfer the presented knowledge into her teaching. This was another frustration for her. "we would do a lattice addition and multiplication and to me that was confusing I don't know if I would wanna go teach the kids that specific method so it was hard".

Most of the stories as a beginning teacher took place in methods course. Only one participant (Liz) was satisfied with her learning in the methods course. The other two participants expressed their frustration as the lack of the mathematical discussions and connection between content and the teaching methods (Emma), and the misguided flow of the course by moving to the more difficult topics before discussing easier topics (Christiana).

Christiana stressed her difficulty in the class due to lack of discussion on easier geometry topics before doing activities with more advanced topics. Even though 3-D shapes would not be considered as advanced topics in geometry, Christiana had difficulty understanding those concepts. "I think more complex level of geometry [3-D shapes such as polyhedra and related vocabulary] is definitely good to teach in college courses but I think you have to start at the basics [2-D shapes and related vocabulary] because not everybody is on the same page". As the order of topics discussed was a concern for Christiana, Emma's concern was the lack of connection between mathematics topics and teaching methods. She expressed that she gathered valuable activities to use in the classroom however she never experienced discussions on those activities. "I prefer to like do some of the mathematics problems and then learn hands on kind of things and have her explain like why she taught us that way or why she did certain things specific".

Thematic analysis findings

From the thematic analyses, three themes, history of geometry learning, perceptions about geometry, effective geometry instructional approaches were emerged. It is important to note that, even though narrative analysis and thematic analysis findings are reported separately, they are embedded in each other. For example, there are both stories as learners and as beginning teacher for participants' perceptions about geometry.

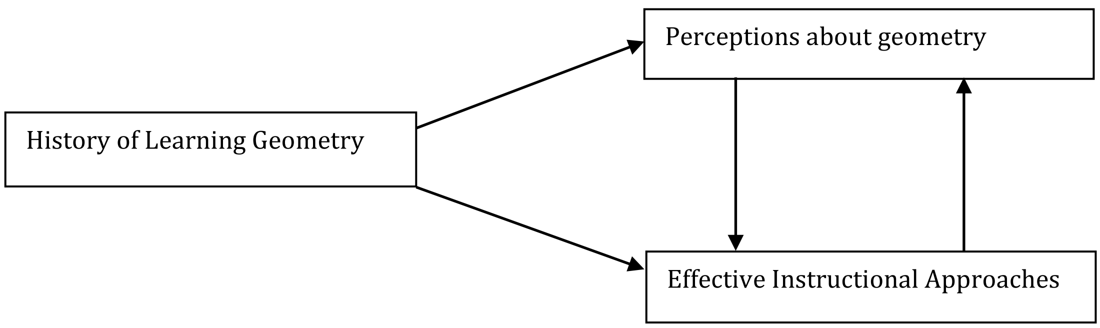

Figure 2. Thematic analysis findings

History of geometry learning. Preservice teachers bring their perceptions, beliefs and learning experiences into the teacher education programs. All three of the participants mentioned how they learned geometry and their teachers especially before college. Their background in geometry played very important role in their learning in college courses especially the methods course. All of them stressed the emphasis on algebraic topics in K-12 education with limited learning opportunities of geometry. They took one geometry course in high school, and they all expressed being dissatisfied with that course. Emma expressed that even though her teacher was "*the easy teacher*" and the teacher did "*fun activities*" she did not like the course. When she was asked about the reasons why she did not like class, she expressed that there were more of the characteristics of a course than having fun to make it effective. Emma brought her geometry perceptions into the methods course, and she expected the instructor to be able to provide content discussions in addition to pedagogical preparation.

Another aspect of participants' history of learning geometry is the focus on algebraic topics in K-12 education. They all perceive geometry as being different than mathematics because they have the perception of mathematics as algebraic topics. Christiana stated that "*I didn't have any clue about geometry [in high school] and then I went to community college and I had to take intro to algebra and then college algebra so it was back to algebra again which algebra is pretty easy I started doing algebra 7th grade middle school so I didn't even think I had to touch*". Their history of learning geometry indeed affects their perceptions about geometry and learning geometry which also reflects on their perceptions about the effective instructional approaches to teach and learn geometry.

Perceptions about geometry. All the participants recognized the importance of visualization in geometry. Participants expressed geometry as a study of shapes and measurement features related to the shapes (such as area). Indeed, the participants gave only 2-D shapes rather than 3-D shapes as examples. For example Christiana thought 3-D geometry as an advanced topic. Some other important topics of geometry such as transformation were not mentioned by any of the participants. Their limited experiences with geometry resulted in distorted perception of geometry. "*for me geometry is basically studying shapes and dimensions and how things fit in things that what I think about geometry*" (Liz).

Furthermore, when participants were asked about effective geometry instruction methods they expressed that effective practices for geometry were different than for the ones for other topics of mathematics. Participants perceived geometry learning different than learning algebraic topics. They preferred to have more real life examples and visual representations for geometry while for other topics learning the formula through direct instruction would be enough.

Effective geometry instruction approaches. The participants addressed the practices and activities which helped their understanding and learning of geometry especially in methods course. The mostly emphasized instruction approach was addressing geometry topics for elementary school (content) before studying instructional aspects of those topics (pedagogical content). Participants stressed their need to study the concepts first in order to be able to understand pedagogical aspects of the topics. Even tough, participants perceived college mathematics courses as reviews before the methods course, because those reviews did not provide desired in-depth geometry understanding for elementary school, they were expecting more content preparation from methods course. As addressed before, only Liz was satisfied from the methods course in terms of receiving both content and pedagogical content preparation. She experienced "*understanding how a child would see it a child cannot grasp this way but he can understand that way*".

All three of the participants addressed practicing content before the pedagogical aspects of geometry. Especially Emma emphasized content preparation because she thought the pedagogical preparation effective yet she had difficulty to grasp the ideas. Emma stated that she could not relate to the activities for elementary school classroom because they discussed only the pedagogical aspect of the activities. "*she [the instructor] gave us a lot of tricks and fun activities and then she actually taught well but she is still I* *guess like besides that it was more like stuff to do in your class we never actually did mathematics problems I prefer to like do some of the mathematics problems and then learn hands on kind of things and have her explain like why she taught us that way or why she did certain things specific*". She wanted to experience the activities as her students in order to be able to understand students process of learning. Even though Christiana experienced content discussions she could not relate the geometry activities to the pedagogical skills. "*we went through a lot of example we used a lot of manipulatives but I don't know a lot of time that's like how to use that in classroom how is this gonna help for future instruction*".

The second aspect of content preparation in the methods course was to progress from easier to more difficult topics in geometry. Christiana's instructor was providing content preparation before the pedagogical discussions, yet she stated that the instruction was not effective in her learning because the discussed geometry topics were advanced for her. All three participants expressed the need to study basic geometry topics (such as 2-D shapes) before advanced geometry topics because they were aware of their limited knowledge of geometry. Christiana especially felt the disproportion because of her limited geometry background. *"[talking about polyhedra and vocabulary for 3-D shapes] I think this is what we went over and that's things I never heard before ... I learned new words like I never heard hexahedrons stuff and I didn't even know what was it six sides 3-D shape never heard some of this stuff in my other geometry class*". Then she stressed the importance of starting from basic in order to address students from different background.

In addition to content preparation in the methods course, the participants addressed some instructional practices that were helpful in their geometry learning. The highly stressed feature of an effective geometry instruction was the use of visual aids such as drawing on the board or on the overhead projector, using of manipulatives such as geoboard. All three of the participants mentioned help of visual drawings in their geometry learning. In the methods course, they experienced geometry manipulatives more than drawings. Especially Liz was very glad to be introduced to the manipulatives in teaching geometry. "*she [the instructor] had the geoboards with rubber band those are really good way of thinking of simpler shapes*".

Another effective instructional practice emphasized by all three of the participants was working in groups. They addressed the supportive feature of group work in classroom activities. Students in groups would explain some topics to each other without asking the instructor. Due to her difficulties with content, Christiana was receiving help from her group members. She could not direct her questions to the instructor so she expressed that "*we do a lot of group work and so there is a lot of interaction going on and that's really helpful*".

Qualitative conclusions

The findings of this investigation may inform mathematics teacher educators on some important issues in preservice elementary teacher education who have limited experience of learning to teach mathematics. Participants of the study took only one mathematics teaching course and there were only two classes (each 3 hours) for geometry teaching. The most important result of this study is participating preservice elementary teachers' lack of geometry knowledge as reported by them. All the participants were very enthusiastic in teaching in elementary school. They all stressed the importance of professionalism to be an effective teacher. They all favour hands-on and meaningful teaching in mathematics. However, they still felt that they were not ready to teach mathematics in elementary school. They expressed that they need to learn more before they began teaching. In other words, good intentions are not enough to be good teachers (Borko et al., 1992). Borko stressed that often teacher education programs do not support preservice teachers in their learning in order to transform them to knowledgeable teachers.

Preservice teachers were aware of their lack of content knowledge. Their limited knowledge in turn affected their learning pedagogical aspects of teaching (Fennema & Franke, 1992). Even though preservice teachers should have been prepared content wise before the methods course, many of them were not equipped with enough content knowledge to focus on pedagogical content preparation. They stressed that content preparation before the methods course was not addressing in-depth understanding for elementary geometry (Ball et al., 2008).

According to participants, the methods course for preservice elementary teachers should provide content knowledge in addition to the pedagogical content knowledge. Even though methods course instructors addressed content, they used different instructional approaches. Among the three participants, only one of them reported an effective integration of content and pedagogy preparation in the methods course. The findings of this investigation stress two important characteristics of studying mathematics content in methods course. First, the mathematics topics should be accessible to the preservice teachers. The difficulty of mathematics topics should be from easier to the more advanced topics. The teacher educators should aim to address the diverse mathematical background that the preservice teachers bring in the classroom. The second characteristic of an effective content preparation in a methods course is to provide the content blended with the pedagogical aspects. In other words, the mathematics content should be addressed in the context of teaching. Participants were aware of that the primary purpose of methods course was not mathematics, but pedagogy. However, without any content discussion the preservice teachers were having trouble relating to the pedagogical examples.

It is important to note that the type of content knowledge that has been asked by participating preservice teachers was not college level mathematics, but mathematics that they would be teaching. They did not feel confident about knowing elementary school geometry for teaching it meaningfully (Browning et al, 2014). This type of knowledge is the type of content knowledge that Ball et al. (2008) called as specialized content knowledge (SCK). In studying SCK, Ball et al. (2008) stressed the importance of using mathematics in the context of teaching because SCK is the mathematics knowledge for only teachers to use in teaching.

Therefore, teacher educators who work in similar setting as in this investigation should address the content needs of preservice teachers in methods courses too. Furthermore, it is important to discuss content in the context of teaching. Compared to their algebra experiences, they have very limited experiences with geometry which results in limited geometry knowledge. In the light of qualitative investigation findings and literature on teacher education, the researchers developed a series of activities to improve elementary school preservice teachers' geometry content knowledge for teaching.

Phase II: Development of Learning for Geometry Teaching Activities

The synthesis of the results from the qualitative investigation, methods course resources such as Van de Walle (2007), and the literature on preservice teacher education yielded to learning for geometry teaching activities on quadrilaterals as an intervention to be used in third phase, quantitative investigation. The findings of participating preservice teachers' experiences in the explained setting were emerged in six principles of activity development.

- *There is a need to address content in addition to pedagogical practices in the methods course.*

- *Preservice teachers' reported their lack of knowledge in 2-D geometry topics especially in quadrilaterals.*
- *Preservice teachers stressed that, in methods course, discussion of content before the discussions of pedagogical practices would improve their learning.*
- *Preservice teachers expressed the importance of the flow of instruction from easier topics to more advanced topics due to various backgrounds among them.*
- *Preservice teachers addressed the effectiveness of using visual aids such as drawings for their geometry learning.*
- *Preservice teachers explained that various forms of activities such as small group works in addition to individual work were helpful in their learning.*

The activities can be grouped in two; geometry activities and pedagogical activity (analyzing student work). These activities will be described below in detail but interested reader may access whole activities from Aslan-Tutak (2009).

Geometry activities. Geometry activities were grouped as: *sorting shapes*, *attributes of shapes*, and *classification of polygons*. The first activity was a sorting activity in which the participants (in pairs) sorted 33 cut-out shapes in groups according to their properties. The groups of shapes were concave, convex, hexagons, pentagons, triangles, quadrilateral, kite, trapezoid, parallelogram, rectangle, rhombus, and square (at least three of each category). When the participants were sorting shapes they experienced defining characteristics of the shapes and the relationship between them. As a result of this activity, the participants worked individually to developed definitions of those shapes.

For the second group of activity (attributes of shapes) participants worked in pairs to study 10 groups of figures. The participants were asked to determine which figure in a group did not belong to others. In other words, the participants had to find a figure which did not share the common characteristics with other three figures. Participants were encouraged to find more than one answer for each group. For example, in a group of four figures, one of them did not belong to others because it was concave while another one did not belong to other three because it was not a quadrilateral. The goal of this activity was for preservice teachers to practice the characteristics of shapes in an open-ended problem solving activity while discussing the relationship between the shapes.

For the last group of activities (classification of polygons) the participants worked in small groups to develop a visual representation (tree diagram) demonstrating the relationships between the polygons especially the quadrilaterals. Participants were given vocabulary (in alphabetical order) to fill the empty spots in the visual representation. The vocabulary were concave, convex, hexagon, kite, parallelogram, pentagon, polygon, quadrilateral, rectangle, rhombus, square, trapezoid and triangle. After the completion of the diagram, participants answered a set of true-false questions based on the diagram. Some of the examples of true-false questions were "All pentagons are regular" and "Only some trapezoids are parallelograms".

In addition to individual characteristics of the activities, the combination of them provided coherence. Participants worked individually, in pairs and small groups. At the end of the each activity, the facilitator led whole class discussions on the topics while providing the right answers. The participants experienced geometry topics with visual representations such as cut-out shapes. Also, the activities progressed through van Hiele geometric thinking levels. Participants began with level 0 and level 1 activities (e.g. sorting) and finished with a level 2 activities (e.g. true-false statements). Therefore, the activities reflected suggestions from both literature and qualitative results.

Analyzing student work. One of the possible designs to provide content in the context of teaching is using student work to analyze what students know and what they are learning.

Using student work has been widely accepted by teacher educators to improve teacher learning and instructional practices (Lampert & Ball, 1998; Little, 2004; McGuire, 2013; Smith 2003). Furthermore, using student work to facilitate teacher learning may result in teachers' deeper content knowledge (Franke & Kazemi, 2001; Kazemi & Franke, 2004). The authors discussed that by analyzing student work, teachers may be forced to think deeply and elaborate on mathematics knowledge while they are trying to understand what students did. "Making sense of children's strategies could be an indirect way for teachers to wrestle with the mathematical issues themselves" (p. 7).

Kazemi and Franke (2004) suggested that the student work to be used in professional development to improve teachers' content knowledge should be challenging in terms of students' errors. With this purpose, the researchers collected student work from elementary schools with mathematically struggling students. As a result of collaboration between the researchers and elementary school teachers, the geometry worksheet for classrooms use was designed. The worksheet consisted of open-ended questions for definitions of some geometry shapes (polygons and quadrilaterals) and 10 figures to be determined if they are certain quadrilaterals with mathematical explanations. To be used in the research, six students' worksheets which were providing most challenging geometry ideas were selected.

During the treatment, the participants were given a protocol to analyze student work. The protocol was developed by suggestions from several resources (E. Kazemi, personal communication, August 17, 2008; NCTM, 2006). First, participants completed the worksheet as students and then they received sample student work. In pairs, the participants discussed what the student did, what the student knew (and misconceptions), what they would ask the student in order to learn more about the student's knowledge of geometry. Then, in small groups (two pairs), participants discussed what they would do to teach these concepts to the student and how they would address the student misconceptions. There were six groups of four participants, and each group discussed a different student's work. For the whole class discussion, the facilitator asked groups to share their student work and their discussions.

Phase III: Quantitative investigation

The third phase of this research is the quantitative investigation which aimed to study effect of using the developed learning to teach geometry treatment on preservice teachers' geometry content knowledge. At the time of implementation, there were three instructors for four sections of the methods course in which one hundred and seven students were enrolled and 102 of them volunteered to participate in the study. All the participants were female. Two of the sections were selected as treatment and other two were selected as control groups. All the instructors were teaching geometry for two weeks (three hour class for each week) during the last third of the semester. Because the focus of this research was geometry, the intervention had to be conducted during the time of geometry instruction of each section. This time restriction is also a rationale of this research. The purpose of the research was to investigate practices that will work for preservice teachers with similar settings and limited opportunities to learn mathematics teaching. Furthermore, as a precaution to avoid researcher bias, another trained instructor delivered the intervention tasks. She was not teaching at the time of this study but she had valuable experience with the student population of this course. One of the researchers was also present in the class during the intervention for observation.

The instrument to measure change in participants' geometry content knowledge, Content Knowledge for Teaching Mathematics Measures (CKT-M Measures)[1], was developed by Learning Mathematics for Teaching (LMT) at University of Michigan. LMT can be seen as continuum of research on mathematics knowledge for teaching (MKT) which was discussed in literature review. The validity and reliability of the instrument was studied by experts from different backgrounds (Ball et al., 2008; Hill et al., 2004, 2008). The instrument addresses the majority of mathematics topics under three categories: number and operations (K-6 and 6-8), patterns functions and algebra (K-6 and 6-8), and geometry (3-8). For the study mentioned in this article, the researchers used only geometry section. Two parallel forms of the geometry section of the test were administered as pre- and post-test. The pre-test consisted of 19 multiple choice questions in 8 stems. The post-test consisted of 23 multiple choice questions in 8 stems.

Quantitative data collection and analysis

Participants completed the CKT-M Measures geometry test one week before geometry instruction. For next two weeks (three hours of instruction for each) they received the geometry instruction and the following week they completed the post-test. In order to address third and fourth research questions (geometry knowledge growth of treatment group and any difference of knowledge growth between treatment and control group) two different analysis methods, repeated measures ANOVA and mixed ANOVA, were used, respectively.

Quantitative results

In order to study geometry knowledge growth of treatment group, repeated measures ANOVA was used. Results showed a significant change in participants' geometry content knowledge, $F(1, 49) = 16.08$, $p<.001$, $R^2 = .25$, eta^2 = .25. This indicates statistically significant positive change in treatment group participants' geometry content knowledge. A mixed ANOVA method of analysis was conducted to study whether there was difference of knowledge growth between treatment and control groups. Results indicated a significant main effect of time $F(1, 91) = 28.38$, $p<.001$ but there was no significant interaction between time and grouping (treatment/control), $F(1, 91) = .21$, $p=.646$. The results showed that geometry knowledge of participants was increased significantly; however the grouping did not have effect on participants' knowledge growth. It can be concluded that even though treatment group participants' geometry content knowledge growth was significant, the difference between treatment group and control group participants' growth in geometry content knowledge was not significant.

Quantitative conclusion

The analysis of growth in treatment group can be interpreted as that use of the activities developed in phase two, from the qualitative investigation, resulted in significant increase in preservice teachers' geometry content knowledge. Even though treatment group participants' increase was more than the increase of control group participants, the difference was not statistically significant. One of the limitations of this investigation to explain non-significant difference between gain scores of participants would be the limited authority in control group instruction. One of the researchers observed the control group instruction. The control group instructor who has certain experience with preservice elementary teachers used an instruction based on elementary school curriculum. Therefore, some common characteristics of these two instructions can be identified as use of learning activities in the context of teaching especially closely linked to the classroom and use of the topic of quadrilaterals. Furthermore, intervention of six hour instruction may not be long enough to provide detectable statistical difference between groups' changes in content knowledge. Because it was not possible to spend more time for geometry in this course, this research can be expended with a similar design for a longer period of time in a different setting.

In a study of middle and secondary school teachers' geometry content knowledge, Fostering Geometric Thinking (FGT), Driscoll and his colleagues used content activities and analysis of student work with in-service teachers (Driscoll et al., 2009). This study showed significant difference between control group teachers who did not receive any professional development and treatment group teachers who received 20-week long intervention. The intervention was designed to provide geometry content experiences for teachers and analysis of student work from teachers own classroom.

Comparison of FGT study and this study reveals other limitations such as selection of the student work. Using student work with preservice teachers might not be as effective as using them with in-service teachers. This study provides a new topic of discussion on using student work with teachers. The effects of using student work might vary in the context of preservice or in-service teacher education. In the case of in-service teachers, participants first experience teaching the materials and then analyze student work. On the other hand, in the case of preservice teachers, participants only experience the materials as a student without teaching them. Therefore, this study might start the discussions such that the role of actual teaching of the materials before analyzing student work.

The results of the study also provide some suggestions for mathematics methods courses. Mathematics teacher educators should consider examining the settings especially the participants and their needs before developing a learning tool for them. For example, one of the highlighted characteristics of the preservice teachers in this setting was limited experience with mathematics and different levels of mathematics preparation among them. The activities provided content discussions before the pedagogical discussions. Also, the activities were in an order to prepare participants to higher thinking levels and more complex parts of the topics.

Discussion

Therefore, as this study provides further understanding on teachers' geometry content knowledge for the particular setting, it also stresses the necessity to study teachers' mathematics content knowledge especially geometry knowledge. This study informs mathematics teacher education in three important areas. First, preservice teachers' reported their limited geometry knowledge as being parallel to previous studies (Jones, 2000; Swafford et al., 1997). Second, for teacher education, learning to teach geometry activities addressing the topics in the context of teaching should be favoured. Instead of knowing factual knowledge of mathematics, teachers should possess specialized content knowledge of mathematics for teaching (Ball et al., 2008). The last but not the least implication of this study is on using student work with teachers. Using student work in the context of preservice and in-service teacher settings might result in different outcomes. In the case of in-service teachers, participants apply the mathematics tasks with students and then analyze their work. On the other hand, in this study, preservice teachers analyzed the student work that collected by the researchers. They never experienced interacting with students about the given mathematics tasks. Even though, it cannot be said for sure about the effect of applying the tasks with students, it is worth to study more about it.

Acknowledgements

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