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Pre-service Teachers' Use of Visual Representations

Gökhan ÖZSOY[1]

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Abstract

The purpose of this research is to evaluate the type and accuracy of the visual representations that the pre-service teachers generate in the pedagogical solution of the word problems. The data on participants' spatial competence levels were obtained through Purdue Spatial Visualization Test and the data on the type of visual representations were obtained by coding the data obtained from the solution of a word problem given to the participants. In the analysis of the data, descriptive analysis and content analysis were applied. As a result of the research, the visual representation types (schematic and pictorial) generated by the pre-service teachers have been determined. The results suggest that the pre-service teachers with high spatial skills can generate the structured schematic representations correctly whereas those with low spatial skills have tendency to generate pictorial representations. Some suggestions for teacher education are presented.

Keywords: Visual representations, spatial ability, problem solving, teacher education

Introduction

Solving mathematical problems is challenging for many primary school students. In classroom problem solving activities, teachers use various visual representations to facilitate the solution of problems. Representations can be thought of as mathematical concepts that act on the senses, as well as external representations of these concepts and representations of cognitive schemas and mathematical ideas acquired through experience (Pape & Tchoshanov, 2001).

Representation; according to Gestalt theory, involves understanding how the parts of a problem come together and constructing a new structure that will help in the process of understanding (Mayer, 1992). Mayer (1985) described representing the problem while constructing the conceptual framework of problem solving as one of the four basic steps of the problem-solving process. In the mathematical problem-solving process, the visuals used in representing the person and objects in the mind related to the problem have an auxiliary function for understanding and solving the problem. According to Kahney (1993), the most important determinant of problem solving is how mental or visual representations are made.

It is thought that the visual representations used in the problem solving process are closely related to the spatial ability. As a matter of fact, the influence of spatial ability in constructing visual representations has been the subject of large amount of research (Blatto-Vallee, Kelly, Gaustad, Porter & Fonzi, 2007; Boonen, van Wesel, Jolles & van der Schoot, 2014; Hegarty & Kozhevnikov, 1999; Lean & Clements, 1981 van Garderen, 2006; van Garderen & Montague, 2003).

Recognition of spatial ability was achieved through an article published by Thorndike (1921). Thorndike has described spatial relevance as the ability to visually express relationships between objects and to understand the functioning of the physical world, and pointed to the necessity of working in this area.

In subsequent studies, Thurstone (1950) described three basic components of spatial ability. These are;

- Mental rotation (the ability to recognize an object when it moves in different directions and angles)
- Spatial visualization (the ability to recognize an object when parts are displaced or completely removed)
- Spatial perception (the ability to use one's body orientation in relation to spatial orientation).

Likewise, Hegarty and Waller (2005) suggested that spatial competence consists of more than one component, while Linn and Petersen (1985) expanded the definitions of the three components previously described by Thurstone (1950) (rotation of mind, spatial visualization, spatial perception). Spatial competence is defined as 'the ability to transform, produce and remember symbolic and linguistic knowledge' (Linn & Petersen, 1985). From this point of view, it is possible to say that visual representations are an expression of the spatial skills of individuals.

The use of visual representations as an effective tool in mathematics depends on the nature of visual representations (Yancey, Thompson, & Yancey, 1989). The nature of the visual representations can be determined by looking at the correctness of the representation of the problem (Lindvall, Tambu-

[1] Ordu University, Faculty of Education, Dept. of Primary Education. Cumhuriyet Campus, 52200, Ordu, Turkey. E-mail: gozsoy@odu.edu.tr
Phone: +90 452 2265200

rino & Robinson, 1982, van Essen & Hamaker, 1990). The accuracy of the representation must include qualifications such as the choice of the appropriate representation and the appropriateness of the problem structure. In this study, the visual representation accuracy was determined on the basis of the characteristics of the visual representation and visual representation of the desired and given requirements.

Purpose of the Research

The purpose of the study is to evaluate the type and accuracy of the visual representations that the pre-service teachers use in the pedagogical solutions of the verbal problems in terms of their spatial skill level.

Research Questions

1. What kind of visual representations do teacher candidates use for the pedagogical solutions of verbal problems?

2. What is the correctness of the visual representations that teacher candidates use in the pedagogical solutions of verbal problems?

3. How does the type and accuracy of visual representations used by pre-service teachers in the pedagogical solutions of verbal problems differ in terms of spatial skill levels?

Importance of study

The role of visual representations in the problem-solving process has been investigated in several studies (Boonen, Reed, Schoonenboom, & Jolles, 2016; Edens & Potter, 2007; van Essen & Hamaker, 1990; van Garderen, 2006; van Garderen, Scheuermann & Jackson, 2012; van Garderen, Scheuermann & Poch, 2014). It is known that the pictorial representations adversely affect the problem solving process. (Boonen et al., 2013, 2014; Hegarty & Kozhevnikov, 1999; van Garderen, 2006; van Garderen & Montague, 2003), that the well-structured schematic representations help to understand the problems, and the steps necessary for computational processes. This clearly indicates the importance of visual representations used by teachers and pre-service teachers because an incorrect or incomplete representation of the teacher may cause confusion in the students. This may have a negative impact on the students' problem solving performance.

The use of appropriate visual representations to support the understanding of the problem in the problem solving process and the improvement of problem solving performance depends on how the teacher configures these visual representations. Today visual representations are mostly used in numerical and fractional teaching in mathematics teaching. However, it is important to remember that visual representations are not only a teaching tool, but a component of the problem-solving process, or even the process itself. The creation of visual representations should not be viewed as an additional aid to problem solving, but as a process developed by students and constituting a part of the problem-solving process. Stylianou (2011) emphasized that visual representations are a process rather than a final product. It is hoped that the current research will contribute to the creation of knowledge and awareness about the type and accuracy of visual representations and to the limited number of studies on primary school and undergraduate mathematics teaching and problem solving.

Method and Study Sample

This study was designed as a descriptive research in the screening model. This method, which is widely used in the field of education, summarizes the characteristics of individuals, groups or environments (Büyüköztürk, Çakmak, Akgün, Karadeniz & Demirel, 2017). The research was conducted with 55 pre-service classroom teachers having taken mathematics teaching courses in the fall semester of 2016-2017 academic year. Participants were identified by convenience sampling method. Thirty six female and 19 male pre-service teachers participated in the current research.

Analysis of Data

Descriptive analysis and content analysis were used in the analysis of the data obtained within the scope of the research. Descriptive analysis (Kümbetoğlu, 2008) refers to direct transfer of the data by being faithful to the original form of the data collected in a qualitative study and by using direct quotations. Content analysis is grounded on the principle of determining some concepts and themes included in the data and to make inferences about the relationships between these themes in addition to the descriptive analysis to reach more exploratory and causal results (Kümbetoğlu, 2008; Yildirim & Şimşek, 2008).

Data Collection and Evaluation Process

A verbal problem which could reveal the pedagogical solutions of the pre-service teachers was created by two field experts. Two field experts were consulted to determine the extent to which the problem created meets the research objectives. The pre-service teachers were given one class hour to produce their pedagogical solutions. During the given period, the pre-service teachers were asked to solve the problem by taking into account the teaching process and the use of appropriate strategies. Following the collection of the research data, the visual representations formed by the pre-service teachers were subjected to content analysis in terms of their accuracy and type. In order to evaluate the accuracy of the visual representations, a representation accuracy form was developed. In order to determine the type of the visual representation, a representation type determination form was developed.

In the process of development of representation accuracy evaluation form and representation type determination form, the relevant literature was examined and the opinions of field experts were sought. After the development of the forms, in order to ensure the reliability of the rater in the content analysis, the representations obtained from the pre-service teachers were separately encoded by the two experts and then agreement was reached between the coders.

Table 1. *Visual Representation Accuracy Evaluation Form*

Items that need to be visually represented in the given problem	Availability (+/-)
Two frogs standing side by side in a lakeshore	
A straight path consisting of 18 lotus leaves	
One of the frogs jumps 3 leaves forward at a time	
The other frog jumps 2 leaves forward at a time	
To mention the common leaves of two frogs until they cross the lake	
Use of units	
Comprehensibility (openness)	
Demonstrating relationships between problem objects	

Determining the Accuracy of a Visual Representation

The problem used in this research: *Two frogs standing side by side on a shore, wanting to reach the opposite shore through a straight path of 18 lotus leaves. One of the frogs can bounce forward three leaves at a time, and the other two leaves forward. How many lilies have both of the frogs stepped on until they cross the lake?*

The visual representation to be created for this problem should include the following properties. The visual representations obtained from the pre-service teachers were evaluated according to this form and the visual representation accuracy score was found.

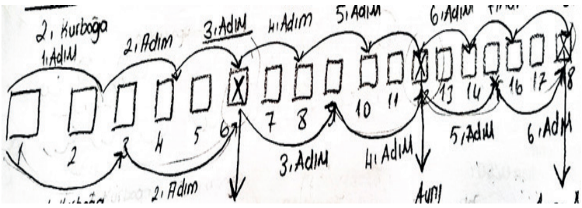

Figure 1. Correctly structured visual representation example

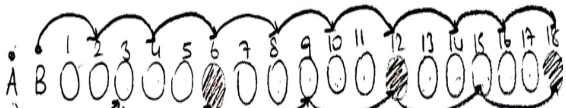

Figure 2. Correctly structured visual representation example

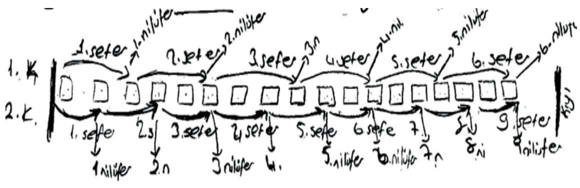

Figure 3. Correctly structured visual representation example

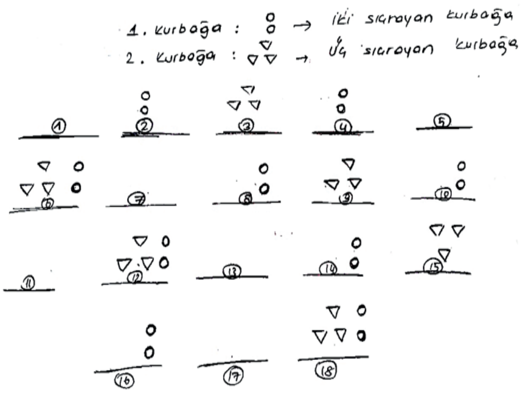

Figure 4. Example of misconfigured visual representation

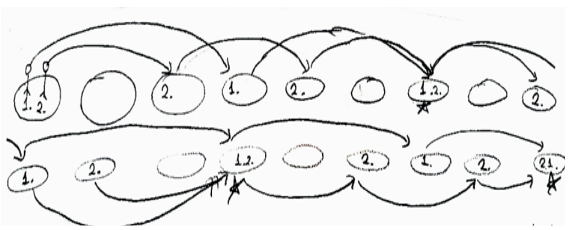

Figure 5. Example of misconfigured visual representation

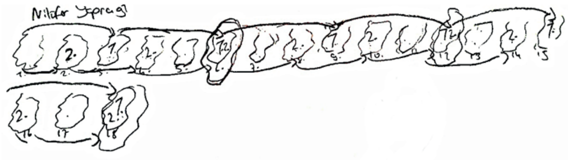

Figure 6. Example of misconfigured visual representation

Determining the Type of a Visual Representation

The 'pictorial' and 'schematic' representations referred as types of visual representation in this research are based on the work of Hegarty and Kozhevnikov (1999). According to this study, visual representations;

- Pictorial representation of the object in terms of shape, colour or brightness;
- The relationship between the parts of the object is a schematic representation of its location and movement on the space.

Table 2. *Visual representation type evaluation form*

Type	Definition
Pictorial Representation	The problem involves the appearance of some random elements of the problem text, without defining the relationships between the items or the necessary calculations. They are visual representations created by neglecting the connections between the spatial relationships and problem objects of the images of persons or objects given in the problem.
Schematic Representation	The spatial relations between the problem parts are specified in a systematic way and the relations between the problem parts and their actions are indicated. Schematic representation can include individually and freely configured drawings as well as mathematics models such as graphs, tables etc.

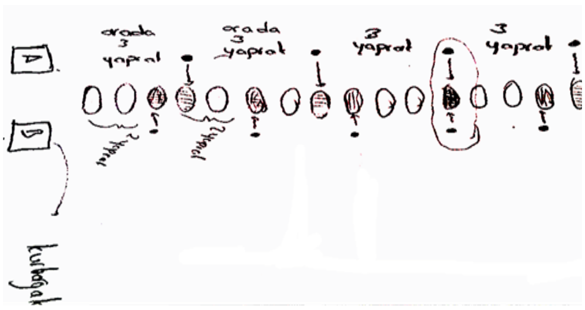

Figure 7. Example of pictorial representation

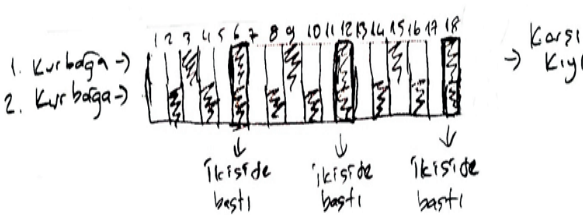

Figure 8. Example of schematic representation

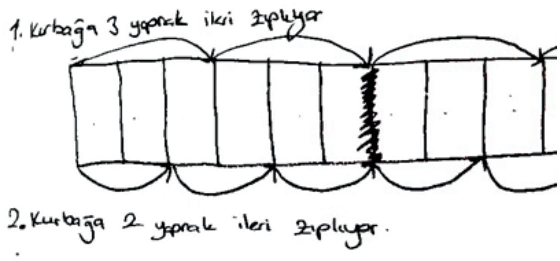

Figure 9. Example of schematic representation

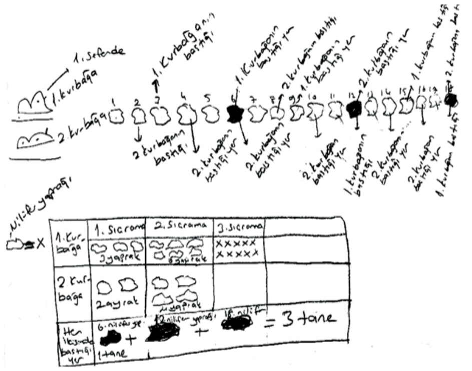

Figure 10. Example of schematic representation

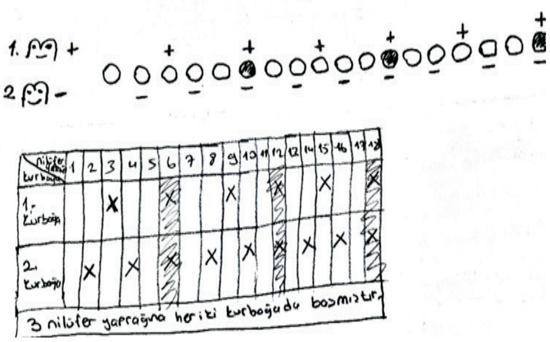

Figure 11. Example of schematic representation

Determination of Spatial Capability Level

The Purdue Spatial Visualization Test (Purdue Spatial Visualization Test) was developed by Guay in 1977 to determine the spatial skill level. The test was adapted to Turkish by Sevimli (2009). The reliability coefficient of the test calculated by split-half method by Sevimli was found to be alpha=0.82; the correlation coefficient between the forms was found to be r= 0.23 and the reliability coefficient calculated with the parallel test method was found to be alpha= 0.88. The reason for the selection of this test for the current research is the abundance of the studies in which the test has been used (Baki & Güven, 2007; Güven & Kösa, 2008; Hacıömeroğlu, 2007) and its compliance with the purpose of the study.

Results

Findings for the First Research Question

What type of visual representation do the pre-service teachers use for the pedagogical solutions of verbal problems?

The pre-service teachers created pictorial and schematic representations. Of the participating teachers, 67.3% created pictorial representation and 32.7% created schematic representations.

Table 3. *Table of visual representations created by the pre-service teachers*

Visual Representation Type		
	Frequency	Percent
Pictorial representation	37	67.3
Schematic representation	18	32.7
Total	55	100

When the table of the visual representations formed by the teacher candidates was examined, it was seen that a large part of the candidates constituted pictorial representation. This implies descriptive drawings without spatial relations and numerical values.

Findings related to the second research question

What is the correctness of the visual representations that the pre-service teachers use in the pedagogical solution of verbal problems?

When the visual representations created by the pre-service teachers were examined in terms of accuracy, it was seen that 14.5% of the participants had one accuracy, 30.9% had four accuracy scores and 5.5% had eight accuracy scores.

Table 4. *Accuracy of the Visual Representations Created by the Pre-service Teachers*

Visual Representation Accuracy		
Accuracy scores	Frequency	Percent
1	8	14.5
2	4	7.3
3	6	10.9
4	17	30.9
5	7	12.7
6	7	12.7
7	3	5.5
8	3	5.5
Total	55	100

It would not be wrong to say that four points and lower scores are inadequate on the basis of this evaluation made out of eight full points in terms of the transfer of what is given in the problem text and the features to be found in the visual representation (clarity, relations, etc.). The fact that the visual representation accuracy of only 5.5% of the pre-service teachers is the exact score is an important finding of the research.

Findings for the Third Research Question

How does the type and accuracy of visual representations used by the pre-service teachers in the pedagogical solutions of verbal problems differ in terms of spatial skill levels?

When the table showing the distribution of the accuracy of representations of the pre-service teachers according to their spatial ability levels was examined, it was seen that the pre-service teachers with a high level of spatial talent did not fall below 6 bands and the pre-service teachers with a low level of spatial talent could not reach 6 bands.

Table 5. *Pre-service teachers' spatial skill level and visual representation accuracy*

Visual Representation Accuracy								
Spatial Capability Level	Scores							
	1	2	3	4	5	6	7	8
Low	7	2	4	7	1	2	0	0
Medium	1	2	2	10	6	4	3	2
High	0	0	0	0	0	1	0	1
Total	8	4	6	17	7	7	3	3

Table 6. *Spatial competence level of the pre-service teachers and the type of visual representation*

Spatial Capability Level	Visual representation type					
	Pictorial		Schematic		Total	
	f	%	*f*	%	*f*	%
Low	20	87	3	13	23	100
Medium	17	56.7	13	43.3	30	100
High	0	0	2	100	2	100

When Table 6 is examined, it is seen that all of the participants with high level of spatial competence created schematic representations. As schematic representations systematic constructs including spatial relationships, they are directly related to spatial capability, and findings support this knowledge. Only 13% of participants with low spatial competence were found to have created schematic representations.

Research findings clearly show that pre-service teachers have difficulty in forming a correct visual representation. Therefore, it is necessary to emphasize how visual representation should be structured in teacher education. Although there are some schematic and correctly structured representations produced by the pre-service teachers, some misconceptions and misuse of some forms of representation have also been explored in the current study.

Little information is available about the visual representations that teachers use in their classroom teaching practices to support the student in understanding the problem. Moreover, the research on whether or not they have enough knowledge about schematic and pictorial representations, how types of visual representations differ from each other, which types of visual representations can support the process, which representations should be avoided is very limited. In order to raise the awareness of the issue, literature should be supported with similar studies on what kind of visual representations should be created during the problem solving process. Such research can be an important contribution to teacher education.

Discussion

Vision is central to us as our biological and socio-cultural being. People have been using images for the recording and communication of information since the cave-painting era. Visual solution to a problem may enable us to engage with concepts and meanings which can be easily bypassed by the symbolic solution of the problem (Arcavi, 2003). Uesaka (2007) studied the factors affecting the use of visual representation and stated that teachers' use of visual representations may positively affect the use of visual representations by students. Given that students are prone to copy the representations used by their teachers in the problem solving process, the importance of the type and accuracy of the representations used by the pre-service teachers becomes clear. The findings of the current research showed that a considerable number of the pre-service teachers created misconfigured and pictorial representations. This finding coincides with the results of the research conducted Boonen, Reed, Schoonenboom, and Jolles (2016) on teachers. They found that visual representations created by teachers were based on personal preferences rather than the expression of the verbal problem in the best possible way. It was seen that after a program was developed to inform teachers, teachers started to create schematic representations instead of misconfigured and pictorial representations. This should be an important issue for teacher training programs and in-service seminars.

This study is a stepping stone for a more comprehensive study to be conducted on teachers. Elicitation of the use of visual representations by teachers through classroom observations and interviews will make an important contribution to the field. In addition, the effectiveness of programs designed as a result of experimental studies to be conducted on teachers and pre-service teachers in terms of helping them create schematic representations can be tested.

The findings of this study suggest that teacher training programs should include applications enabling pre-service teachers to construct their visual representations accurately and to develop their spatial abilities.

Some studies addressed the issue of how to overcome difficulties experienced by students in the problem solving process through the instructional programs entailing the use of visual representations (Jitendra, 2002; Montague, 2003; Montague, Warger, & Morgan, 2000). Mathematical representations are rooted in visuospatial thinking and develop through visual experience (Amalric, Denghien, & Dehaene, 2018). It would be very important to provide effective experiences to children through school activities. In this sense teachers have important duties and therefore it is necessary to include visual presentations in teacher education. It is also believed that the provision of complementary applications such as programs and seminars that will enable the development of pre-service teachers' spatial skills and visual representations can improve the quality of teacher education in Turkey.

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