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Is There a Place for Proof in Primary School Mathematics? A Comparison of Türkiye's 2018 and 2024 Curricula

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Abstract

This study compares the developmental components of proof in Türkiye's 2018 and 2024 primary school mathematics curricula. Using comparative document analysis, it examines the learning objectives in the 2018 curriculum and the learning outcomes and process components in the 2024 curriculum. The data were analyzed using the framework (Hemmi, Lepik, & Viholainen, 2013), which comprises proof, argumentation, investigation, structure, definition, and logic. The findings indicate that neither program includes statements directly targeting proof; however, the 2024 curriculum demonstrates a stronger and more pronounced orientation toward thinking processes related to proof compared to the 2018 curriculum. While the invesitgation (mostly problem-solving and estimation) and structure components were prominent in the 2018 curriculum, the 2024 curriculum places greater emphasis on recognizing mathematical structures and relationships, as well as on reasoning, evaluation and justification-based argumentation. An examination of the process components also supports this trend. Although the 2024 curriculum does not explicitly address proof, it offers broader opportunities for students to identify mathematical relationships, formulate generalizations, draw conclusions, and provide justifications—skills that can serve as a foundation for their future learning of proof.

Keywords:

Developmental Proof, Document Analysis, Mathematics Curriculum, Primary School Mathematics, Proof

Introduction

Proof is not merely a formal and abstract activity specific to advanced mathematical topics; it is one of the fundamental components of mathematical reasoning (Hanna, 2000; Stylianides, 2007). In the process of learning mathematics, students are expected to go beyond routine procedures to understand how mathematical knowledge is generated, why mathematical results are valid, and on what grounds these results are based (Harel & Sowder, 1998; Stylianides, 2007; Otten, et al. 2014). In this context, proof and the process of proving are central to students' abilities to formulate mathematical claims, justify these claims, evaluate different solution paths, and develop valid arguments (Thompson et al., 2012).

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In school mathematics, the term "proof" often evokes symbolic, deductive arguments grounded in axioms and definitions. This view can make proof appear to be an activity reserved for secondary and higher education (Selden & Selden, 1995). However, proof is not merely a tool for testing validity; it is also a powerful way to make sense of mathematical concepts, discover relationships, and justify one's thinking (Hanna, 2000; Stylianides, 2009). Proof at the primary school level should be defined not as an approach that views students as "little mathematicians" imitating the formal language of adult mathematicians at an early age (Hiebert et al., 1996), but rather as a coherent reasoning activity appropriate to the class's current level of knowledge and representation (Stylianides, 2007).

At the primary school level, proof can take the form of a student explaining why a mathematical claim is true, justifying the strategy used, formulating a generalization that goes beyond specific examples, and demonstrating—through appropriate representations—why this generalization holds for all relevant cases. According to Stylianides (2007), a proof is a mathematical argument consisting of interconnected statements that establish the truth or falsity of a specific mathematical claim. This argument consists of statements accepted by the classroom community, valid forms of reasoning accessible to students, and representational tools appropriate for the grade level. This approach allows proof to be expressed in different forms according to students' developmental levels without compromising its mathematical validity. Consequently, the general justifications developed by primary school students using verbal explanations, diagrams, concrete materials, tables, patterns, and arithmetic properties can also be evaluated as proof activities when they meet the appropriate conditions. From this perspective, primary school is not too early to introduce proof; it is a formative period for developing the foundations of proving. Recognizing patterns, making conjectures about mathematical relationships, testing them against examples, explaining why a result holds, and evaluating peers' justifications can prepare students for later work with formal proof. Stylianides (2007), for example, showed that third-grade students could justify general mathematical claims using mathematical tools and representations appropriate to their level. The aim of primary school mathematics instruction, therefore, is to engage students in questions such as "Why is this result true?", "Does this method always work?", and "How can we show that it holds for all cases?"

However, the fact that proof and reasoning are considered fundamentally important in curricula does not mean that students are provided with sufficient learning opportunities. In an analysis of fifth-grade mathematics textbooks, Bieda et al. (2014) found that, on average, only 3.7% of tasks offered opportunities for reasoning and proof. This finding demonstrates that even when justification and proof are specified as general goals in curricula, they are often reflected only to a limited extent in lesson content and mathematical tasks. Similarly, studies conducted at various grade levels show that proof-related activities are often limited to specific topics and that students struggle to generate authentic justifications (Thompson et al., 2012; Zhang & Qi, 2019).

Although curricula and textbooks may provide limited opportunities for students to engage in proof-related reasoning, such opportunities are important for developing the ability to generalize, justify, and verify mathematical claims (Davis, 2012; Zhang & Qi, 2019). On the other hand, since the 2000s, many countries have begun to include "proof" (proof/reasoning) and "justification" (justification/argumentation) components in their mathematics curricula. For example, Hemmi et al. (2013) showed that the Estonian and Finnish curricula address proof-related competences from the early grades, whereas the Swedish compulsory curriculum addresses them only weakly and locates proof mainly at upper secondary level. International discussions of mathematics curricula have noted the growing prominence of proof in curriculum documents across several educational systems (Hanna et al., 2009).

Intended curricula are foundational documents that define the scope and direction of the mathematical experiences envisioned for students. Including actions such as "explains," "justifies," "generalizes," "draws conclusions," "relates," "verifies," and "evaluates" in the curricula can create important opportunities for students to engage in thinking processes related to proof. However, expectations regarding proof are not always expressed directly through the concepts of "proof" or "proving." Expressions such as mathematical reasoning, justification, pattern recognition, hypothesis formation, generalization, relating mathematical structures, and evaluating the validity of results can also point to different dimensions of developmental proof (Hemmi et al., 2013; Thompson, 2014). Therefore, curricula must be examined not only in terms of explicit proof statements but also in terms of the implicit components that prepare for and support proof.

Hemmi et al. (2013) examined mathematics curricula for grades 1–12 in Estonia, Finland, and Sweden and identified both explicit and implicit references to competencies related to proof. Adopting a developmental view of proof, they analyzed curriculum statements using six categories: proof, argumentation, investigation, structure, definition, and logic. This framework is particularly useful for examining primary school curricula because it captures not only explicit references to proof but also opportunities to investigate, generalize, justify, and reason about mathematical structures—processes that may support students' later learning of proof.

A substantial body of research focuses on reasoning and proof tasks in textbooks or on opportunities for proving at the middle and secondary school levels (Bieda et al., 2014; Thompson et al., 2012; Zhang & Qi, 2019). While there are studies examining national curriculum standards (Davis, 2014; Hemmi et al., 2013), research that comparatively examines how curriculum changes have transformed learning opportunities related to proof at the primary school level is more limited. In Türkiye, the inclusion of process components alongside learning outcomes in the 2024 mathematics curriculum calls for closer examination of proof-related actions such as justifying, making connections, generalizing, and drawing inferences.

This study aims to compare the learning objectives, learning outcomes, and process components of Türkiye's 2018 and 2024 primary school mathematics curricula (grades 1–4) from the developmental perspective on proof proposed by Hemmi et al. (2013). In doing so, it seeks to identify the scope of proof-related learning opportunities articulated in the two curricula. Rather than focusing solely on whether the term "proof" appears explicitly, the study examines the extent to which and the ways in which the curricula address forms of thinking that may prepare students for proving in later years. Accordingly, the study addresses the following research question:

What proof-related components are represented in the learning objectives of Türkiye's 2018 primary mathematics curriculum and in the learning outcomes and process components of the 2024 curriculum?

How do the two curricula differ in the ways they articulate proof-related mathematical actions, particularly argumentation, investigation, and structural thinking?

Conceptual Framework: Developmental Proof and Proof-Related Competencies

The developmental proof approach acknowledges that proof competencies are not skills that suddenly emerge at a specific grade level; rather, they develop gradually alongside students' mathematical experiences. From this perspective, proving involves interconnected processes such as investigating, identifying patterns, formulating conjectures, generalizing, explaining, justifying, and evaluating arguments (Hemmi et al., 2013). Although students' language, representations, and forms of reasoning change over time, the effort to establish why a mathematical claim is true remains relevant across grade levels.

Stylianides' (2007) understanding of proof, structured according to the classroom community, provides an important foundation for applying the developmental approach to proof in primary school. For an argument to count as proof, it must draw on statements accepted by the classroom community, use mathematically valid forms of reasoning, and employ representations accessible to students. A primary school student, for example, may justify a general property of numbers using manipulatives, drawings, or verbal explanations. However, merely checking a few examples or relying on the teacher's authority is not accepted as proof of a general claim.

This perspective also requires a distinction between proof and activities that prepare students for proving. Identifying patterns and formulating conjectures, for example, do not in themselves constitute proof, but they are important parts of the proving process. Similarly, asking students to explain why a result is true may not elicit a complete proof, but it can encourage reasoning grounded in justification. The framework developed by Hemmi et al. (2013) allows these different types of curriculum statements to be examined within a shared conceptual framework while retaining distinct analytical categories.

Hemmi et al.'s Six-Category Framework

The framework developed by Hemmi et al. (2013) classifies curriculum statements related to developmental proof into six categories. These categories do not represent six sequential stages; rather, they describe distinct but interconnected dimensions of proving. Although a program statement may be related to more than one dimension depending on its context, in content analysis, the dominant meaning of the statement and the fundamental mathematical action expected of the student can be taken into account.

Proof

The "Proof" category encompasses statements in the curriculum that explicitly include the concepts of "proof," "proving," or "deductive reasoning." Hemmi et al. (2013) address expectations under this category, such as students learning specific proofs, participating in proof-based activities, and understanding the role of proof in mathematics. It is common for the term "proof" not to appear directly in primary school curricula. However, this does not mean that the curriculum contains no content that could serve as a foundation for proof. The limited presence of explicit proof statements in primary school further highlights the importance of categories such as argumentation, investigation, and structure within the developmental approach to proof.

Argumentation

Argumentation encompasses concepts such as mathematical reasoning, explanation, justification, drawing conclusions, verification, and evaluating the validity of a solution or idea. Although the term "proof" is not used explicitly, students are expected

to connect a mathematical claim with reasons that support it. At the primary school level, expectations such as "explains the strategy used," "justifies why a result is correct," "communicates an inference," or "evaluates different solution methods" may fall within this category.

Argumentation is important to developmental proof because it asks students to explain why an answer is correct, rather than merely produce it. However, not every explanation constitutes a mathematical argument. For a statement to be classified in this category, the expected explanation should include a justification grounded in mathematical relationships, properties, or operations.

Investigation

Investigation involves searching for patterns and relationships, exploring mathematical situations, formulating conjectures, comparing examples, generalizing, and justifying the resulting generalizations. Hemmi et al. (2013) regard investigation as a generative aspect of proving: mathematical claims to be proved often emerge from examining examples, noticing patterns, and conjecturing possible relationships.

In primary school mathematics, this category may include extending number and shape patterns, identifying a pattern rule, examining changes in the results of operations, and generalizing from observations. Finding only the next term of a pattern, however, may not be sufficient to constitute investigation. A stronger opportunity for developmental proof arises when students are encouraged to examine the underlying relationship, formulate a rule or conjecture, and explain why it holds.

Structure

Structure encompasses curriculum statements concerned with understanding connections among mathematical concepts, operations, representations, and properties. Proving requires students not only to make logical inferences but also to recognize the mathematical structure about which they are reasoning. Understanding operations as related mathematical structures, rather than as isolated rules, can help students develop more general justifications.

At the primary school level, this category may include recognizing the inverse relationship between addition and subtraction, connecting repeated addition with the equal-groups interpretation of multiplication, relating place value to the structure of numbers, and examining geometric shapes through their defining properties. Structural thinking provides an important foundation for proving because it directs students' attention beyond individual examples to the general mathematical relationships underlying them.

Definition

Definition involves the competencies necessary for understanding and using mathematical definitions and distinguishing their defining characteristics. Mathematical proofs are based on what the concepts used mean and which properties are necessary for those concepts. Therefore, definitions should be viewed not merely as terms to be memorized, but as the foundation for mathematical classification and reasoning.

In primary school, students' ability to classify geometric shapes based on defining characteristics—such as sides, vertices, or angles—rather than their appearance is related to this category. For example, when discussing whether a square is also a rectangle, a student's reliance on the defining characteristics of shapes rather than their prototypical appearances is an indication of definition-based reasoning. Such experiences lay the foundation for understanding the role of definitions in proofs in later years.

Logic

Logic encompasses program statements related to the precision of mathematical expressions, the validity of inferences, conditional relationships, and the consistency of arguments. Within the framework developed by Hemmi et al. (2013), this category also includes the clear and logical justification of the conclusions reached.

At the primary school level, logic can be observed in the ability to distinguish between true and false statements, establish "if... then..." relationships, evaluate whether a claim is always true or only true in certain cases, and find counterexamples that refute a general claim. It is also important for students to distinguish the meanings of expressions such as "all," "some," and "none" in order to understand the scope of mathematical generalizations. While these activities do not constitute the teaching of formal logic, they form the foundation for valid mathematical inferences.

In this study, the framework developed by Hemmi et al. (2013) is used as an analytical tool to identify learning opportunities related to proof in the 2018 and 2024 primary school mathematics curricula. The primary reason for choosing this framework is that it encompasses not only explicit proof statements but also the developmental precursors to proof—namely, justification, investigation, generalization, structural thinking, the use of definitions, and logical inference.

However, the inclusion of a component in the curriculum does not necessarily mean that the relevant skill is taught in the classroom or acquired by students. While the curriculum outlines intended learning opportunities, how these opportunities are

implemented in the classroom depends on teachers' interpretations, textbooks, the tasks used, and assessment practices (Cai & Cirillo, 2014; Thompson, 2014). Therefore, the findings of this study should be interpreted not as students' actual proof-taking competencies, but as the proof-related learning opportunities offered or envisaged by the 2018 and 2024 curricula at the primary school level.

Method

Research Design

In this study, a comparative document analysis was conducted to examine the primary school mathematics curricula published in Turkey in 2018 and 2024 in terms of their components related to proof. Document analysis is a research method that involves the systematic selection, examination, coding, and interpretation of written documents in accordance with specific research questions (Bowen, 2009). Since the statements in the curricula were examined according to predefined theoretical categories, deductive content analysis was used in this study.

Frequency and percentage values were also utilized to compare the appearance of the categories identified through qualitative content analysis across the curricula. In this regard, the study is a comparative document analysis that qualitatively classifies the meanings and functions of program statements and quantitatively describes the resulting classifications based on the program, data unit, and grade level.

Hemmi et al. (2013) cautioned that differences in the level of generality of curriculum documents make purely quantitative text analysis uninformative. In line with this caution, the frequencies and percentages reported here are used only to describe the general orientation of each document; no inferential comparison between the two curricula is made, and the units of analysis (2018 objectives; 2024 outcomes; 2024 process components) are not treated as equivalent. Unlike the original frame, in which a single statement can be related to several categories, each unit here received one code; this decision is justified and its consequences are discussed under limitations.

In this study, the curricula were examined at the intended/written curriculum level. Consequently, the statements related to proof identified in the curricula were evaluated not as indicators of instruction occurring in the classroom or of the competencies students achieved, but rather as the learning opportunities the curricula were intended to provide to students (Cai & Cirillo, 2014; Thompson, 2014).

Data Sources

The data sources for this study were the 2018 and 2024 mathematics curricula (Ministry of National Education [MoNE], 2018, 2024) published by the Ministry of National Education of Türkiye. The study was limited to the primary grades, and only the sections covering grades 1–4 were examined. No sampling was undertaken; all learning objectives, learning outcomes, and process components within this scope were included in the analysis.

Because the two curricula differ in structure, the units of analysis were defined according to each curriculum's terminology. In the 2018 curriculum, the intended learning is expressed as learning objectives. Each objective is assigned a code, such as "M.4.2.1.2," whose letter and numbers indicate, respectively, the subject, grade level, learning area, sub-learning area, and objective number. Each learning objective was treated as a separate unit of analysis.

The 2024 curriculum distinguishes between learning outcomes and the process components associated with each outcome. Each learning outcome has a code, such as "MAT.1.1.1," indicating the subject, grade level, learning area number, and outcome number, respectively. Process components are listed beneath the relevant outcome as items labeled a, b, c, and so forth. In total, 673 units were analyzed: 229 learning objectives from the 2018 primary mathematics curriculum, and 111 learning outcomes and 333 process components from the 2024 curriculum (Table 1).

Table 1

Number of units analyzed in the 2018 and 2024 primary mathematics curricula (grades 1–4)

Curriculum	Unit of analysis	Number of units
2018 primary mathematics curriculum (1-4. Grade)	Learning objective	229
2024 primary mathematics curriculum (1-4. Grade	Learning outcome	111
2024 primary mathematics curriculum (1-4. Grade	Process component	333
Total		673

In the 2024 curriculum, learning outcomes and process components are presented as distinct elements. Each learning outcome and each process component was therefore examined as a separate unit of analysis. Headings for themes and learning areas, explanatory text, implementation suggestions, and assessment guidance were excluded. The analysis was limited to learning objectives, learning outcomes, and process components that directly state the mathematical actions expected of students.

Analytical Framework and Coding Procedure

The data were analyzed using the framework developed by Hemmi et al. (2013) to examine competencies related to developmental proof in mathematics curricula. The framework organizes

proof-related curriculum statements into six categories: proof, argumentation, investigation, structure, definition, and logic. These categories were treated as distinct but interconnected dimensions of developmental proof, rather than as sequential stages. Units that could not be associated with any of the six categories were coded as containing no proof-related component. This allowed all units to be classified and the proportion without an identifiable proof-related component to be determined for each curriculum. Coding was based on the meaning of each statement, rather than the presence of particular keywords alone. For each unit, the expected mathematical action, the content to which that action referred, and the meaning of the statement as a whole were considered together. For example, the verb "identifies" does not, on its own, indicate a category; what students are expected to identify and for what purpose also matter. A statement asking students to identify the rule of a pattern could be coded as investigation, whereas one asking them to identify a relationship between operations could be coded as structure.

During the coding process, two researchers reviewed the framework of Hemmi et al. (2013) and specified the scope and distinguishing features of each category, as well as the conditions under which a statement would be excluded from it. They then prepared a coding guide containing category definitions, decision rules, and examples selected from the curricula. Statements from both curricula and different grade levels were subsequently used in pilot coding. During the pilot coding process, potential overlaps between the "argumentation–investigation," "investigation–structure," and "structure–definition" categories were specifically evaluated. Statements that the researchers coded differently were discussed, and the decision rules in the coding guide were clarified. Finally, the learning objectives in the 2018 curriculum and the learning outcomes and process components in the 2024 curriculum were coded using the coding guide. Each analysis unit was assigned a single code representing the statement's predominant mathematical function. In cases where a statement could be associated with more than one category, the primary action expected of the student and the statement's purpose within the curriculum were taken as the basis for coding. For example, if a statement is expected to involve generalization following pattern investigation, the "investigation" code was selected; if explaining why the derived result is valid takes precedence, the "argumentation" code was chosen. In the 2018 curriculum, learning objectives that ask students to solve problems were coded as Investigation because their stated emphasis is on working toward a solution. Checking a solution, where explicitly required, was also treated as part of this investigative process. This is a broad reading of Hemmi et al.'s Investigation category (finding regularities, formulating conjectures, generalizing), because routine problem solving and estimation do not necessarily involve these actions; a sensitivity analysis is therefore reported in the Findings. These objectives do not explicitly ask students to justify why a generalization is valid or to evaluate its validity using mathematical examples. In contrast, the 2024 curriculum specifies such an action in a process component of MAT.4.2.7. This explicit expectation to evaluate the validity of a generalization was the basis for coding the 2024 learning outcome as Argumentation, even though several of its process components were coded as Investigation. Statements that did not contain a clear expectation regarding any proof component were coded as "containing no components."

Finally, when coding the learning outcomes in the 2024 curriculum, the assessment was not based solely on the wording of the learning outcome itself; rather, a holistic evaluation was conducted that included the process components listed under the relevant learning outcome. Accordingly, all process components associated with the learning outcome were first examined individually, and then the predominant demonstration component generally targeted by the learning outcome was determined. In other words, the final code for a learning outcome was determined not only based on the verbal statement of the outcome but also on the mathematical actions and common orientation revealed by the process components that explain how this outcome is achieved. The aim was to prevent any loss of context that might arise from a brief or general statement of the learning outcome and to define the targeted proof component more comprehensively. For example, although the learning outcome "MAT.4.2.7. Ability to solve problems requiring the four basic arithmetic operations" includes four Investigation, two Structure, and only one Argumentation-coded process component, the learning outcome itself was coded as Argumentation. For example, the learning outcome "MAT.4.2.7. Solving problems requiring the four operations" was coded as argumentation, although four of its process components were coded as investigation, two as structure, and only one as argumentation. When the components were considered together, the researchers judged argumentation to be the outcome's overarching aim (see Table 4). On the other hand, for the learning outcome "MAT.2.2.5. Ability to reason about the results of multiplication and division," a single process component was coded as "Structure," and no component was coded as "Argumentation"; however, because the learning outcome includes the ability to reason, it was coded as "Argumentation." In the coding process for the process components, each component was coded independently of the others.

The holistic procedure described above was applied only to the 2024 learning outcomes; the 2018 objectives, which have no sub-components, were

coded on their wording alone. Learning outcomes and objectives were therefore coded on different bases, and this asymmetry may partly explain why the proportion of units without a proof-related component is much lower among 2024 learning outcomes (15.3%) than among 2018 objectives (59.8%). In two cases (MAT.4.2.7 and MAT.2.2.5) the outcome-level code was determined by a minority component or by the verb of the outcome rather than by the predominant function of its components.

Validity and Interrater Agreement

To establish the reliability of the coding process, the curriculum data were independently coded by two researchers. The coding focused on the process components and learning outcomes included in the primary-school mathematics curriculum and separately on the learning objectives included in the 2018 mathematics curriculum. A total of 444 coding units for the 2024 curriculum were independently evaluated by the two coders using six categorical codes (Argumentation, Definition, Investigation, Logic, Structure, and None). For the 2018 curriculum, the analysis was conducted across all 229 independently coded units using five categorical codes (Argumentation, Definition, Investigation, Structure, and None). Because the coding categories represented nominal classifications, interrater agreement was assessed using Cohen's kappa coefficient (Cohen, 1960), which was preferred over simple percentage agreement because it accounts for the agreement that may occur by chance.

For the 2024 curriculum, the interrater agreement analysis revealed that the two coders agreed on 353 of the 444 coding units, corresponding to an observed agreement of 79.50%, against an expected chance agreement of 26.89%. Cohen's kappa was calculated as $\kappa = .720$, 95% *CI* [.669, .770], *SE* = .027, $p < .001$, indicating a statistically significant and substantial level of interrater agreement. This result suggests that the two researchers applied the coding scheme consistently across the curriculum data. The kappa value was interpreted cautiously, however, given that kappa coefficients are sensitive to the distribution of categories and should therefore be considered alongside the observed percentage agreement and the broader coding context (Landis & Koch, 1977; McHugh, 2012). According to the benchmark criteria proposed by Landis and Koch (1977), a kappa value in the .61–.80 range corresponds to substantial agreement. Taken together, these findings indicate that the coding framework applied to the learning outcomes and process components of the 2024 primary-school mathematics curriculum demonstrates high objectivity and robust reliability, rendering the resulting dataset suitable for subsequent empirical evaluation.

For the 2018 curriculum, the interrater agreement analysis showed that the two coders assigned the same code to 168 of the 229 coding units, corresponding to an observed agreement of 73.36%, against an expected chance agreement of 40.72%. Cohen's kappa was calculated as $\kappa = .551$, 95% CI [.457, .633], *SE* = .044, $p < .001$. According to the Landis and Koch (1977) benchmark criteria, a kappa value in the .41–.60 range corresponds to moderate agreement, indicating a moderate—though not near-perfect—level of interrater agreement. Coding discrepancies were subsequently reviewed by the researchers, and consensus was reached through discussion before the final dataset was established. Overall, this result indicates that the two researchers applied the coding scheme to the 2018 Mathematics Curriculum in a consistent, though somewhat less robust, manner compared to the 2024 curriculum.

Ethics Statement

This study did not involve human participants, animals, personal data, or the collection of data from individuals. The study was based exclusively on the systematic analysis and comparison of publicly available official mathematics curriculum documents published by the Ministry of National Education of Türkiye. Therefore, ethical approval from an institutional ethics committee was not required. All sources used in the study were appropriately acknowledged and cited in accordance with academic and publication ethics principles.

Results

This study examined the learning objectives, learning outcomes, and process components in Türkiye's 2018 and 2024 primary mathematics curricula for grades 1–4 in terms of their proof-related components. The analysis included 229 learning objectives from the 2018 curriculum and 111 learning outcomes and 333 process components from the 2024 curriculum. Table 2 presents the distribution of these units across the analytical categories, providing an overview of each curriculum's orientation toward proof-related thinking.

Table 2

Distribution of proof-related components in the 2018 and 2024 primary mathematics curricula

Proof components	2018 learning objectives	2024 learning outcomes	2024 process components
Proof	0 (0.0)	0 (0.0)	0 (0.0)
Argumentation	2 (0.9)	28 (25.2)	29 (8.7)
Investigation	51 (22.3)	15 (13.5)	46 (13.8)
Structure	35 (15.3)	49 (44.1)	95 (28.5)
Definition	4 (1.7)	0 (0.0)	18 (5.4)
Logic	0 (0.0)	2 (1.8)	11 (3.3)
No proof	137 (59.8)	17 (15.3)	134 (40.2)
Total	229 (100)	111 (100)	333 (100)

Table 2 shows that neither the 2018 learning objectives nor the 2024 learning outcomes and process components contained a statement coded as proof. Nevertheless, other components that may support the development of proof were more prominent in the 2024 curriculum. The proportion of units coded as containing no proof-related component was 59.8% among the 2018 learning objectives, compared with 15.3% of the 2024 learning outcomes and 40.2% of the 2024 process components. Because the 2024 learning outcomes were coded holistically on the basis of their process components, the most comparable contrast is between the 2018 learning objectives (59.8%) and the 2024 process components (40.2%), the most fine-grained units available in each curriculum.

Of the 229 learning objectives in the 2018 curriculum, 2 (0.9%) were coded as argumentation, 51 (22.3%) as investigation, 35 (15.3%) as structure, and 4 (1.7%) as definition. None were coded as proof or logic. Thus, proof-related statements in this curriculum were concentrated primarily in investigation and structure.

Of the 111 learning outcomes in the 2024 curriculum, 28 (25.2%) were coded as argumentation, 15 (13.5%) as investigation, 49 (44.1%) as structure, and 2 (1.8%) as logic. None were coded as definition. The learning outcomes were therefore concentrated particularly in structure and argumentation.

Among the 333 process components in the 2024 curriculum, 29 (8.7%) were coded as argumentation, 46 (13.8%) as investigation, 95 (28.5%) as structure, 18 (5.4%) as definition, and 11 (3.3%) as logic. Structure was also the most frequent category among the process components.

The following sections present the findings for each proof-related category.

Argumentation

While the 2018 primary school curriculum included 2 learning objectives related to argumentation, 2024 primary school curriculum includes 28 learning outcomes and 29 process components. Table 3 lists the two learning objectives related to argumentation in the 2018 primary school curriculum.

Table 3

Learning objectives related to argumentation in the 2018 primary mathematics curriculum (grades 1–4)

Learning objectives
M.3.3.7.1. Explains the necessity of standard liquid measuring instruments and units, and performs measurements using liters or half-liters.
M.4.2.3.3. Measures angles using non-standard units and explains the necessity of standard units of measurement.

The two learning objectives coded as argumentation in the 2018 mathematics curriculum ask students "to perform measurements while explaining why measurement is necessary" and "to explain the need for units of measurement". Table 4 presents selected learning outcomes and process components related to argumentation in the 2024 curriculum.

Table 4

Learning outcomes and process components related to argumentation in the 2024 primary mathematics curriculum (grades 1–4)

Proof component	Learning outcome or process component
Argumentation	MAT.1.1.8. Estimate the lengths and masses of objects using appropriate nonstandard measuring tools.
No proof	a) Identifies a nonstandard measuring tool appropriate for the length or mass to be measured.
Investigation	b) Estimates the measurement results in terms of the selected nonstandard unit.
Argumentation	c) Evaluates the accuracy of the estimate.
Argumentation	MAT.2.2.5. Reason about the results of multiplication and division.
No proof	a) Identifies the components of multiplication and division operations.
Structure	b) Identifies relationships among the components of multiplication and division operations
No proof	c) States the results of estimates and mental calculations involving multiplication and division.
No proof	ç) Explains the results of of estimation and mental calculations involving multipliaction and division.
Argumentation	MAT.3.1.14. Estimate the durations of events and make judgments about them
Structure	a) Relates the durations of events to one another.
Argumentation	b) Draws inferences based on experiences of how long events take.
Argumentation	c) Evaluates the inferred durations in relation to the actual durations.
Argumentation	MAT.4.2.7. Solve problems requiring the four operations.
Structure	a) Identifies the given and required information by interpreting the problem.
Structure	b) Identifies relationships among the operations required by the given and required information.
No proof	c) Identifies the given information and converts it into appropriate mathematical representations.
No proof	ç) Explains the problem in their own words after representing it mathematically
Investigation	d) Estimates the result and develops strategies for carrying out the operations.
No proof	e) Applies the selected strategy or strategies to solve the problem.
Investigation	f) Checks the solution methods and changes a strategy that does not lead to a solution.
Investigation	g) Reviews the strategies used or developed and evaluates possible shortcuts.
Investigation	ğ) Generalizes the types of problems to which successful strategies can be applied.
Argumentation	h) Evaluates the validity of the generalization using mathematical examples.

Table 4 presents one learning outcome associated with argumentation from each grade level, together with its process components. MAT.1.1.8 was coded as argumentation because it expects students to evaluate the accuracy of their estimates. MAT.2.2.5 was coded as argumentation because the learning outcome explicitly calls for reasoning about the results of multiplication and division. MAT.3.1.14 was assigned to this category because students are expected to draw inferences from their experiences and evaluate those inferences against actual durations. MAT.4.2.7 was coded as argumentation because one of its process components asks students to evaluate the validity of a generalization using mathematical examples.

An examination of the actions expressed in the 28 learning outcomes coded as argumentation in the 2024 primary mathematics curriculum shows an emphasis on making inferences, solving problems, reasoning, making decisions, estimating, reflecting, evaluating, and making judgments.

Table 5

Argumentation-related actions in the learning outcomes and process components of the 2024 primary mathematics curriculum

Learning outcome actions
Making inferences (2); solving problems (3); reasoning (5); making decisions (4); estimating (9); reflecting (1); evaluating (2); making judgments (2)
Process component actions
Evaluating (10); making inferences (3); making judgments (13); articulating consistency (3)

The process components coded as argumentation involve evaluating, making inferences, making judgments, and articulating consistency.

Notably, none of the argumentation-coded actions in Table 5 is explicitly "justify" or "prove"; most concern estimating, evaluating, and making judgments. The only explicit expectation to test a generalization is the process component "Evaluates the validity of the generalization using mathematical examples" (MAT.4.2.7). The data therefore support the claim that the 2024 curriculum emphasises evaluation and judgment more than the justification of general claims.

Investigation

In the 2018 curriculum, 51 learning objectives were coded as investigation. In the 2024 curriculum, 15 learning outcomes and 46 process components were assigned to this category. Table 6 presents one illustrative learning objective from each grade level in the 2018 curriculum.

Table 6

Learning objectives related to investigation in the 2018 primary mathematics curriculum (grades 1–4)

Learning objectives
M.1.2.3.1. Identifies the rule in a pattern of objects, geometric solids, or shapes and completes the pattern by determining its missing elements.
M.2.3.1.4. Estimates lengths in metres or centimetres and checks the estimate by comparing it with the measured result.
M.3.2.2.1. Determines by folding a shape that it has more than one line of symmetry.
M.4.1.2.4. Solves problems requiring addition with natural numbers

The Table 6 presents one learning objective associated with investigation from each grade level. M.1.2.3.1 involves identifying a pattern rule and determining missing elements; M.2.3.1.4 involves making an estimate and checking it against a measurement; and M.3.2.2.1 involves identifying lines of symmetry by folding a shape. These actions were classified as investigation. M.4.1.2.4, which asks students to solve problems requiring addition with natural numbers, was also assigned to this category on the interpretation that problem solving involves selecting and applying a suitable strategy. Taken together, the examples illustrate activities such as exploring patterns, testing estimates, drawing conclusions from concrete experiences, and solving problems.

Table 7

Investigation-related actions in the learning objectives of the 2018 primary mathematics curriculum

Solves problems (32); estimates, compares the estimate with the result of a calculation, and checks its accuracy (17); identifies a pattern rule and completes the pattern (1); identifies a line of symmetry by folding (1).

An analysis of the 51 learning objectives related to investigation in the 2018 curriculum reveals that the vast majority of them pertain to problem-solving and prediction activities.

Table 8 presents examples of learning outcomes and process components coded as investigation in the 2024 curriculum. Because no Grade 1 learning outcome was coded directly as investigation, one learning outcome was selected from each of Grades 2–4.

As shown in Table 8, MAT.2.1.4 expects students to observe forward and backward skip counting, identify a pattern, and then generalize it. The learning outcome was coded as investigation because generalizing requires students to move from particular cases toward a broader relationship. In MAT.3.3.5, students are expected to draw on their experience

with standard units of capacity to estimate a quantity of liquid. Estimation was coded as investigation because it involves applying prior knowledge to a new situation and proposing a possible result. Students then compare their estimate with the measured result, allowing them to test its accuracy. In MAT.4.3.10, the process of developing a coding strategy was interpreted as investigation.

As shown in Table 9, the 15 learning outcomes coded as investigation in the 2024 curriculum involve skip counting, synthesizing structures or models, estimating, generalizing, solving problems, and developing strategies or structures.

Table 8

Learning outcomes and process components related to investigation in the 2024 primary mathematics curriculum (grades 1–4)

Proof-related component	Learning outcome or process component
Investigation	MAT.2.1.4. Skip count forward and backward.
No proof	a) Makes observations about forward and backward skip counting on a hundred chart.
Structure	b) Identifies patterns when skip counting forward and backward by twos up to 20, by threes up to 30, by fours up to 40, and by fives up to 100.
Investigation	c) Generalizes the pattern identified while skip counting forward and backward.
Investigation	MAT.3.3.5. Estimate quantities of liquid in standard units of capacity.
Structure	a) Relates prior experience with standard units of capacity to the quantity of liquid to be estimated.
Investigation	b) Estimates the quantity of liquid in the selected standard unit of capacity.
Structure	c) Compares the estimate with the measurement results and makes a judgment about its accuracy.
Investigation	MAT.4.3.10. Develop coding strategies to construct a structure based on geometric shapes.
Logic	a) Establishes logical relationships between knowledge of geometric shapes and structures.
Investigation	b) Develops a coding strategy based on the relationships identified.

Table 9

Investigation-related actions in the learning outcomes and process components of the 2024 primary mathematics curriculum

Learning outcome actions
Constructing (7); skip counting (2); synthesizing structures/models (2); estimating (2); generalizing (1); and solving problems (1)
Process component actions
Making estimates (9); formulating/developing strategies (6); changing strategies (3); generalizing (5); constructing structures/models (4); forming an integrated whole based on relationships (4); posing problems (1); establishing relationships (3); interpreting (3); formulating conjectures (2); testing (2); evaluating shortcuts (3); and checking (1)

In the process components, prominent actions include estimating, formulating conjectures, testing conjectures with examples, generalizing patterns, developing problem-solving strategies, changing strategies when necessary, evaluating solution methods, constructing original structures or models, and checking solutions. These findings show that the 2018 learning objectives coded as investigation were concentrated largely on problem solving and estimation, whereas the 2024 curriculum expresses investigation through more detailed process components.

Structure

In the 2018 curriculum, 35 learning objectives were coded as structure. In the 2024 curriculum, 49 learning outcomes and 95 process components were assigned to this category. Table 10 presents one illustrative learning objective from each grade level in the 2018 curriculum.

Table 10

Learning objectives related to structure in the 2018 primary mathematics curriculum (grades 1–4)

Learning objectives
M.1.1.2.3. Recognizes that changing the order of the addends does not change the sum.
M.2.2.3.1. Completes a repeating geometric pattern by identifying the missing elements.
M.3.3.4.1. Shows the relationship between the lira and the kuruş.
M.4.1.4.2. Shows that changing the order in which three natural numbers are multiplied does not change the product.

Table 10 presents one learning objective associated with structure from each grade level. M.1.1.2.3 was coded as structure because it asks students to recognize that the sum remains unchanged when the addends are reordered. M.2.2.3.1 was assigned to this category because completing the missing elements requires students to use the structure of a repeating pattern. M.3.3.4.1 concerns the relationship between the lira and the kuruş, while M.4.1.4.2 asks students to show that changing the order in which factors are multiplied does not change the product. Overall, these objectives emphasize recognizing and expressing invariance, patterns, and relationships among units or operations.

Table 11 presents selected expressions associated with structure in the learning objectives of the 2018 primary mathematics curriculum.

Table 11

Structure-related actions in the learning objectives of the 2018 primary mathematics curriculum

Recognizes that the result remains unchanged/identifies relationships/recognizes the meaning/recognizes the relationship/recognizes/understands (9); explains similarities and differences/explains what it means/explains the relationship, converts one into the other/explains the relationship, expresses one in terms of the other/shows the relationship/expresses (16); establishes a relationship/relates (2); completes a pattern/creates a pattern (7); orders events using reference points (1).

The learning objectives and their key expressions indicate that the 2018 curriculum primarily emphasizes recognizing and explaining mathematical structures, establishing relationships, and creating patterns.

Table 12 presents selected learning outcomes and process components coded as structure in the 2024 curriculum.

Table 12

Learning outcomes and process components related to structure in the 2024 primary mathematics curriculum (grades 1–4)

Proof-related component	Learning outcome or process component
Structure	MAT.1.1.2. Analyze relationships among parts when counting a group of objects arranged either irregularly or in an orderly way.
No proof	a) Identifies parts of a group of objects arranged either irregularly or in an orderly way.
Structure	b) Identifies relationships among the parts when counting the group of objects.
Structure	MAT.2.2.3. Interpret the relationship between addition and subtraction.
Structure	a) Examines the relationship between addition and subtraction.
No proof	b) Converts addition and subtraction operations into their inverse forms.
Definition	c) Restates the relationship between addition and subtraction.
Structure	MAT.3.1.3. Order numbers.
Structure	a) Identifies numbers up to 1,000 according to their preceding and succeeding relationships.
No proof	b) Groups numbers up to 1,000 according to the tens and hundreds to which they belong.
No proof	c) Classifies numbers up to 1,000 according to the nearest ten and hundred.
No proof	ç) Expresses numbers up to 1,000 in terms of the nearest ten and hundred.
Structure	MAT.4.1.4. Structure forward and backward skip counting.
Structure	a) Establishes hierarchical relationships when skip counting forward and backward.
No proof	b) Draws on their knowledge of skip counting to reach a target number by counting forward or backward in hundreds and thousands.

Table 12 presents one learning outcome associated with structure from each grade level. MAT.1.1.2 asks students to analyze relationships among the parts of a group of objects, supporting an understanding of the whole through part–whole relationships. MAT.2.2.3 focuses on examining the relationship between addition and subtraction and recognizing that they are inverse operations. In MAT.3.1.3, identifying numbers according to preceding and succeeding relationships draws attention to the ordered structure of numbers. MAT.4.1.4 was coded as structure because it asks students to establish hierarchical relationships while skip counting forward and backward. Together, these examples show that the category encompasses identifying and interpreting mathematical structures involving part–whole relationships, inverse operations, number order, and hierarchical patterns.

As shown in Table 13, the most frequent actions in the learning outcomes coded as structure are analyzing and interpreting. These are followed, in descending order of frequency, by classifying, distinguishing, ordering, comparing, and structuring.

Table 13

Structure-related actions in the learning outcomes and process components of the 2024 primary mathematics curriculum

Learning outcome actions
Analyzing (23); interpreting (15); classifying (5); distinguishing (2); ordering (2); comparing (1); structuring (1)
Process component actions
Identifies relationships/relates/establishes relationships/establishes connections (59); examines a relationship (12); groups by properties (6); lists similarities/differences/examples (4); analyzes (3); compares with a criterion (2); converts one into the other (2); identifies a pattern (2); recognizes a relationship or necessity (2); reviews a need (1)

The 2024 process components express a wider range of actions related to structure than the learning objectives in the 2018 curriculum. These include identifying and examining relationships, connecting ideas, grouping objects by their properties, listing similarities and differences, analyzing, comparing against a criterion, converting between forms, identifying patterns, recognizing relationships or the need for a particular approach, and reviewing that need.

Definition

Only four learning objectives in the 2018 primary mathematics curriculum were coded as definition. In the 2024 curriculum, no learning outcome was assigned to this category, but 18 process components contained definition-related expressions. Table 14 presents two illustrative learning objectives from the 2018 curriculum.

Table 14

Learning objectives related to definition in the 2018 primary mathematics curriculum (grades 1–4)

Learning objectives
M.1.2.1.1. Classifies and names geometric shapes according to their numbers of vertices and sides.
M.3.1.6.2. Divides a whole into equal parts and identifies each part as a unit fraction.

Of the four learning objectives coded as definition in the 2018 curriculum, two involve classification, one involves classifying and naming, and one involves identifying. Although their stated actions differ, all four concern the properties used to define mathematical objects. Three objectives in geometry ask students to classify shapes by their numbers of vertices and sides or by their side lengths. The remaining objective concerns the identification of a unit fraction.

Table 15 presents the actions expressed in the process components coded as definition in the 2024 curriculum.

Table 15

Definition-related actions in the process components of the 2024 primary mathematics curriculum

Process component actions
Expresses/restates (8); classifies (5); names (4); explains in their own words (1).

As shown in Table 15, "expressing/restating" was the most frequent action among the process components coded as definition. In statements such as "Restates the relationship between addition and subtraction" and "Expresses the meanings of equality in the context of the four operations," students are expected to articulate mathematical concepts and relationships clearly. The next most frequent actions were "classifying" and "naming." Process components such as "Classifies geometric shapes" and "Names geometric shapes" involve grouping mathematical objects by shared properties and identifying them with appropriate terms. The component "Explains the terms vertex, face, and edge in their own words" asks students to express mathematical meanings using their own language. Overall, the actions coded as definition focus on naming and classifying concepts and expressing mathematical relationships in different ways.

Logic

No learning objective in the 2018 curriculum was coded as logic. In the 2024 curriculum, two learning outcomes and 11 process components were assigned to this category. Table 16 presents the two learning outcomes from the 2024 curriculum.

Table 16

Logic-related actions in the learning outcomes and process components of the 2024 primary mathematics curriculum

Proof-related component	Learning outcome or process component
Logic	MAT.2.3.6. Select appropriate strategies for reaching a destination, taking distance and direction into account.
Structure	a) Establishes criteria for reaching the destination.
No proof	b) Gathers information relevant to the criteria established.
Logic	c) Generates options based on the information collected and the criteria established.
Logic	ç) Logically evaluates the options generated.
No proof	d) Selects an option that meets the criteria.
No proof	e) Adjusts the process when the criteria change.
Logic	MAT.3.3.7. Construct a symmetrical shape from a given part using a line of symmetry.
Logic	a) Examines the given part of the symmetrical shape and identifies logical relationships.
No proof	b) Completes the shape using a horizontal or vertical line of symmetry.

The learning outcomes in Table 16 were coded as logic because their process components ask students to identify logical relationships or evaluate options through logical reasoning.

As shown in Table 17, 11 process components in the 2024 curriculum were coded as logic.

Table 17

Logic-related actions in the process components of the 2024 primary mathematics curriculum

Process components
Establishes hierarchical, causal, or logical relationships (9); checks logical consistency (1); generates options based on a criterion (1).

Nine of the 11 process components coded as logic ask students to establish hierarchical, causal, or logical relationships, as illustrated by the statement "Establishes hierarchical, causal, or logical relationships in problem situations involving the four operations." Of the remaining two components, one asks students to check logical consistency, and the other asks them to generate options based on an established criterion.

Discussion, Conclusions, and Implications

The key finding of this study is that proof components in the 2024 primary school mathematics curriculum are addressed in a more visible and detailed manner compared to the 2018 curriculum. Although no statement falling directly under the "Proof" category

was found in either curriculum, the proportion of statements containing no proof components decreased from 59.8% in the 2018 learning objectives to 15.3% in the 2024 learning outcomes In addition, 59.8% of the process components in the 2024 curriculum were associated with a proof-related category (the complement of the 40.2% coded as "no proof"; this figure coincides numerically with, but is conceptually unrelated to, the 59.8% "no proof" figure of 2018). These findings suggest that the 2024 curriculum gives greater attention to actions such as evaluating and making judgments, examining relationships, identifying and constructing patterns, and, to a more limited extent, generalizing and justifying (with the caveat that these differences partly reflect differences in the coding units).

Hemmi et al. (2013) regard such actions as foundations for proving in later years. Their analysis of the Finnish curriculum identified proof-related components from the early grades, including drawing justified conclusions, observing regularities, establishing relationships among mathematical structures, classifying, and making simple logical evaluations. Stylianides and Stylianides (2022) suggest that introducing students to proof involves helping them recognize why mathematical claims require justification and develop a working understanding of what makes an argument a proof. From this perspective, the greater emphasis on explaining, making inferences, and making judgments in the 2024 curriculum may provide a foundation for students to develop a sense of the need for proof. Research on proof in primary school likewise emphasizes explaining why mathematical claims hold, generalizing from patterns, evaluating counterexamples, and supporting conclusions with justifications acceptable to the classroom community, rather than relying on formal symbolic representations (Campbell et al., 2020; Stylianides, 2007).

Structure was the most prominent category in the 2024 curriculum, accounting for 44.1% of learning outcomes and 28.5% of process components. Actions such as analyzing part–whole relationships, examining connections between operations, and constructing patterns may help students recognize mathematical relationships beyond individual calculations. In the developmental framework of Hemmi et al. (2013), understanding mathematical connections and structures is a component of proof-related competence. The prominence of structure in the 2024 curriculum may therefore provide a foundation for later justification and proof. However, recognizing a mathematical relationship does not in itself constitute a proof; the recognized structure must be generalized, and its validity in every case must be justified. At this point, the increase in the argumentation component in the 2024 curriculum takes on greater importance. While the argumentation category remained quite limited at 0.9% in the 2018 curriculum, its rise to 25.2% in the 2024 learning outcomes indicates that student activities such as evaluating, estimating, explaining, drawing conclusions, making judgments about validity, and defending solutions are now more explicitly targeted. This finding is consistent with the trend in international curricula that proof is structured through argumentation and justification, particularly at younger ages (Campbell et al., 2020). However, it is noteworthy that the proportion of argumentation has dropped to 8.7% among the process components. It can be argued that the expectation of reasoning and judgment—emphasized at the learning outcome level—has not been translated to the same extent into acts of justification in the sub-processes that explain how this outcome is achieved. In other words, while the program strongly emphasizes argumentation at the general objective level, it provides more limited guidance on how students should support a claim with evidence, evaluate another person's reasoning, or improve a weak argument.

These findings can be considered alongside those of Zhou et al. (2026), who examined number and early algebra tasks in Chinese primary mathematics textbooks for Grades 1–6. They reported that 17.18% of the tasks offered opportunities for reasoning and proof, with expressing justifications being the most prominent activity. They also identified a developmental shift from recognizing patterns in the lower grades toward formulating conjectures in the upper grades, while noting a weak connection between conjecture formulation and justification. A similar issue may be considered in relation to Türkiye's 2024 curriculum. Although investigation, structure, and argumentation are represented, they do not always appear as connected steps through which students identify a pattern, formulate a conjecture, justify it, and evaluate the justification. Galiç et al. (2025) likewise emphasize the need to view proof-related activities as an integrated progression connecting pattern identification, conjecture formulation, the development of arguments that do not yet constitute proofs, and proof construction.

The findings for investigation need to be considered in both quantitative and qualitative terms. This category accounted for 22.3% of the 2018 learning objectives, compared with 13.5% of the 2024 learning outcomes and 13.8% of the 2024 process components. At first glance, these lower proportions might suggest fewer opportunities for investigation. However, most of the 51 learning objectives coded as investigation in the 2018 curriculum concerned problem solving and estimation. The 2024 process components encompass a wider range of actions, including formulating and testing conjectures, generalizing patterns, developing and revising strategies, constructing structures or

models, and checking solutions. Thus, although investigation represents a smaller proportion of the units analyzed in 2024, it is articulated through a more varied and detailed set of processes. This finding highlights the need to consider the mathematical actions represented within a category alongside its frequency. In their analysis of fifth-grade textbooks, Bieda et al. (2014) found that only 3.7% of tasks offered opportunities for reasoning and proof. They found no tasks requiring students to evaluate whether a proposed justification was valid or convincing, and reported that many justification tasks relied on empirical examples rather than general arguments. Platz (2022) similarly found that primary school students often used empirical justifications, but could develop valid justifications through examples representing a general structure and thought experiments when supported by suitable tasks and teaching. These findings suggest that primary school students can engage in proving, but instruction needs to address the distinction between checking that a claim holds for several examples and using an example to explain why it holds generally. From this perspective, process components in the 2024 curriculum that ask students to "evaluate the validity of a generalization using mathematical examples" or "test a conjecture" are valuable. Classroom work with these components should make clear that examples can help students investigate a claim, while checking a few cases does not establish its general validity. Aricha-Metzer and Zaslavsky (2019) showed that an example becomes productive for proving when students recognize the general mathematical structure it represents. Accordingly, statements in the 2024 curriculum such as "evaluates the accuracy of an estimate," "draws an inference," and "identifies relationships" are more likely to become meaningful opportunities for proof-related learning when teachers ask: "Why does this always hold?", "How does this example represent all relevant cases?", and "Can you find a counterexample?"

The limited visibility of the definition and logic components is another aspect of the curriculum that needs to be strengthened in terms of developmental proof. While there are no direct learning outcomes in the definition category in the 2024 curriculum, 5.4% of the process components are coded under this category; in the logic category, 1.8% of the learning outcomes and 3.3% of the process components are coded under these categories. The processes of definition, classification, and naming form the common conceptual foundation upon which mathematical argumentation rests, particularly in primary school. Similarly, logical verification and the identification of causal relationships enable students not only to construct their own arguments but also to evaluate whether an argument is valid. Lehmann and Friend's (2025) systematic review demonstrates that mathematical argumentation is not merely a cognitive activity but a cognitive and social process that requires students to express the connections between claims, evidence, and reasoning both orally and in writing. Therefore, rather than implicitly distributing the components of definition and logic across other categories, it may be beneficial to link them more explicitly to the processes of argument construction and evaluation.

Finally, the results of this study highlight the potential learning opportunities that the 2024 program offers students; they do not provide direct evidence of how these opportunities are implemented in the classroom. It cannot be assumed that a learning opportunity in the program will be reflected in the same way in written materials, teacher practice, assessment, and student learning (Cai & Cirillo, 2014; Thompson, 2014). In particular, how phrases such as "reason," "make connections," or "identify logical relationships" are concretized through a task, a teacher's question, and a student's product is crucial. The findings of Bieda and colleagues (2014) show that general curriculum goals can translate into limited learning opportunities in textbook tasks. Therefore, to assess the potential of the 2024 curriculum in terms of developmental proof, it is necessary to examine the alignment between textbooks, teacher guides, in-class tasks, and exam questions. Galiç et al. (2025) note that it is also necessary to examine how reasoning and proof tasks included in the written curriculum or textbook are interpreted and implemented by teachers. Similarly, the systematic review by Stylianides et al. (2024) reveals that a significant portion of proof research focuses on the student–content relationship; there is a need for more holistic studies that consider teachers, students, and content together. Therefore, while the expansion of the argumentation and structure components in the 2024 curriculum represents significant progress, the extent to which this expansion contributes to students' proof-based thinking will be determined by teacher guidance, the tasks used, textbooks, and classroom norms regarding justification.

In conclusion, although the 2024 primary school mathematics curriculum does not directly aim to teach proof, it provides a stronger foundation for developmental proof—through recognizing mathematical structures, explaining relationships, making inferences, and judging the validity of conclusions—compared to the 2018 curriculum. These findings should, however, be interpreted in light of several limitations. First, the research examines only official curriculum documents and does not provide data on classroom practices. Second, assigning a single code to each unit of analysis to represent its dominant mathematical function partially limits the structure of an expression, which can contain multiple proof components simultaneously. Third, the learning objectives in the 2018 curriculum and the

learning outcomes and process components in the 2024 curriculum are not at the same level of detail. Therefore, frequencies and percentages indicate the general orientations of the two curricula but do not constitute a direct comparison between fully equivalent data units. Fourth, category frequency does not, on its own, indicate the quality or cognitive level of the learning opportunity provided. Coding a statement as argumentation or investigation does not guarantee that the student will construct a general and valid mathematical argument in the classroom.

Finally this study analysed only the units that directly state the mathematical actions expected of students: the learning objectives of the 2018 curriculum and the learning outcomes and process components of the 2024 curriculum. General aims, explanatory text, and curriculum-level skill or competence statements were deliberately excluded, even though Hemmi et al. (2013) also examined the general aims of the curricula they analysed. This decision reflects how fundamentally the two documents differ in approach and structure. The 2018 curriculum is organised mainly around learning areas and objectives, with relatively limited supporting material. The 2024 curriculum is built on a different conceptualisation of learning. Alongside learning outcomes and process components, it includes a substantial amount of additional material, such as skills, dispositions, and values, as well as explanatory and implementation-related text. These elements have no counterpart of comparable status or function in the 2018 document. Including them would have meant comparing sets of statements that differ in kind, scope, and purpose, and any resulting frequencies or percentages would have reflected the architecture of each document rather than its orientation toward proof. Moreover, such statements are typically formulated at a general level (e.g., reasoning or problem solving across the curriculum) and are not tied to specific mathematical content, which makes it difficult to determine what students are expected to do in relation to a particular mathematical claim. For these reasons, we limited the comparison to the units whose function is comparable across the two documents, namely the statements specifying what students are expected to do in mathematics.

Data Availability

The data analyzed in this study consist of publicly available official mathematics curriculum documents published by the Ministry of National Education of Türkiye. These documents are publicly accessible and are cited in the manuscript. No individual-level or confidential data were collected or analyzed.

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