

# Russian Mathematics in Norwegian Classrooms Challenges in Implementing Zankov's Didactic-Methodological System in Primary Mathematics Education in Norway

Gregorios Daniel Schevig Brogstad<sup>a</sup>

883

<sup>a</sup> Correspondance Details: Gregorios Daniel Schevig Brogstad, Østfold University College Department of Natural Sciences, Practical-Aesthetic, Social and Religious Studies, Norway.

E-mail: gregorios.brogstad@hiof.no

ORCID: https://orcid.org/0009-0008-2342-3109

## Abstract

Developmental Education in Mathematics (DEM) according to Zankov's model, or "Russian mathematics", has been in use in Norway since 2009 in an increasing number of schools (about 100 elementary schools in 2024). There has been relatively little research into the implementation of this teaching method in a Norwegian context. In this article we review the fundamental elements and characteristics of Vygotsky's Zone of Proximate Development (ZPD) and the didactic principles and methodology in Zankov's system. We also discuss some challenges concerning the implementation of Zankov's didactic system in Norwegian primary schools and some possible explanations.

### Keywords:

Vygotsky, Zankov, Devlopmental Education, ZPD, Mathematics, Didactic Principles, Methodology, Primary School

### Introduction

/hat is optimal teaching for students? How is a knowledge system formed in the minds of schoolchildren by systematic learning? What do connections between didactic principles, the content in education, and teaching methods look like? What exactly is the influence of education on the mental, emotional, and physical development of students? Questions like these raise several important and interesting issues. Many teachers, searching for the "golden key" that opens the doors to optimal teaching, want a practical answer to these questions and not just theoretical considerations. The relationship between education and development is not easy to uncover because "... any pedagogical influence can only lead to a result other than through the student's mind, i.e., by triggering this or that mental activity in the student" (Zankov, 1973a, 128, author's translation). Cultural-historical psychology has undoubtedly become the most influential theory of mental development with Vygotsky as the most cited psychologist, with his concept of "the zone of proximate" development" (ZPD), the golden zone of teaching. Since 2009 Norwegian teachers have been experimenting with



www.iejee.com ISSN: **1307-9298** 

2025 Published by KURA Education & Publishing. This is an open access article under the CC BY-NC- ND license. (https://creativecommons.org/ licenses/by/4.0/)



Zankov's teaching method, which has its own didactic system and philosophy built on Vygotsky's theories: "... we proceed from the theoretical principles of L. S. Vygotsky" (Zankov, 1963b, 12, author's translation). Developmental education in mathematics (DEM) has attracted great interest among several teachers. But how is DEM and Vygotsky's and Zankov's concept of the ZPD implemented in Norwegian classrooms? Some preliminary research into the implementation of the Zankov system in Norway has been done (Gjære & Blank, 2019). Research by Gjære (2022, 2023) may indicate certain challenges even for teachers with strong loyalty to Zankov's didactic principles. These findings should be properly addressed, and possible explanations searched for.

# Vygotsky and Zankov: Zone of Proximate Development (ZPD)

Lev Semyonovich Vygotsky (1895-1934), "the Mozart of psychology" (Toulmin, 1978, 51-52), revolutionized psychological science (Newman & Holzman, 1993) in that he created new approaches in cognitive psychology. Among other things, Vygotsky, popularly called the "founder of the sociocultural learning theory" (social constructivism), studied the relationship between teaching-learning (обучение, obuchenie) and children's development (развития, razvitiya) both a psychological and pedagogical problem - and concluded that "The only good form of teaching is the one that leads development ..." (Vygotsky, 2001, 167, author's translation). Zankov agrees: "Learningteaching (обучение) comes ahead of development (развития)" (Zankov, 1968, 12, author's translation). Teaching and education are the motor of the child's cognitive development. Vygotsky's famous but undercommunicated "theorem" reveals his thinking: "One step in learning may result in a hundred steps in development" (Vygotsky, 1982 apud Zaretsky, 2016, 149; cf. Zankov 1963b, 22).

The well-known concise and "classical" definition of the ZPD from 1935 (Zaretsky, 2021) of

the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers

appeared in Vygotsky's first English translation (Vygotsky, 1978, 86, 131). If the goal of education is to promote the student's development, the teaching in the classroom should address not what is manifestly achieved (the actual level of development), but the abilities, mental functions, and skills that a child has neither expressed nor achieved as yet (the proximate or potential level of development). These are developed with the help of the teacher or other more capable fellow students who function as "collaborators" for the student. What the child can do today with the help of adults, it will be able to carry out tomorrow on its own. In this way the zone of proximate development will help us to define tomorrow's achieve-ments and the dynamics of the child's development, taking into account not only what it has already mastered, but also its process of growth (Vygotsky, 2017, 366)

The zone of proximal development defines those [higher mental] functions that have not yet matured but are in the process of maturation, functions that will mature to-morrow but are currently in an embryonic state. These functions could be called "buds" or "flowers" of development rather than the fruits of development. (Vygotsky, 1978, 86)

If the goal of education is to promote the development of the student, "The role of teaching as the source of development [is] creating the zone of proximate development ..." (Vygotsky, 2017, 368). While the zone of actual development (ZAD) talks about development that has already taken place, the zone of proximate development (ZPD) speaks of future development. Vygotsky's texts indicate that he placed greater emphasis on the ZPD than the 1935 definition would suggest. The ZPD, that exists because of the presence of maturing mental functions, is often presented in a simple way like this:

### Figure 1

ZAD, ZPD and ZND



To uncover processes that can tell us about the students' development, we need a "system" (Vygotsky, 1982, apud Zankov, 1968, 20) due to the principle of systematic learning where the "system is a whole that represents a unity" (Zankov, 1968, 86, author's translation). Zankov's didactic system operationalizes the ZPD concept in classroom teaching (Guseva & Solomonovich, 2017; Zankov 1963b; 1975/1977a). The essence of proximate development and Zankov's system is that the teaching must address the student's mental abilities as they are emerging, but which have not yet matured. By following Vygotsky's theoretical standpoint, Zankov considered collective learning activity, either with adults or peers, as the core of the ZPD:

This level [the ZPD] is identified by noting the kinds of problems that the child is unable to cope with by himself, but can solve with the aid of grownups, in collective activity, or through imitation. But what a child can do in cooperation with someone else today, he will be able to do alone tomorrow (Zankov, 1977a, 18)

336

With good collaboration between teacher and student, the ZAD can be expanded. "Training creates a zone of nearest development, which then turns into actual development" (Zankov, 1963b, 11-12, author's translation). At the same time, Zankov believes that the ZPD is not the only place where development can take place! Collaboration and imitation are not the only conditions for development as Vygotsky believed. Zankov writes: «... the zone of proximate development ... does not represent the only way of learning to influence children's development» (Zankov, 1963b, 12, author's translation, cf. Longarezi & Ferola, 2023, 16-17). The point is that the teacher can also organize the material so that the students process it in their own independent way, without help: "... in the process of independently solving the questions posed, the child advances in one or another area of mental activity" (Zankov, 1963b, 13, author's translation). The "collaboration" is done in advance. Perhaps we can call this the ZID, the zone of independent development, and place it as a separate zone within the ZPD and ZAD, wherein "imitation is completely excluded" (Zankov, 1963b, 13, author's translation) and the children perform problem solving on their own. The ZPD interacts with other ways that training influences the development of mental activity of schoolchildren. There is a difference between independent and assisted task performance.

#### Figure 2

ZPD in Vygotsky (1978) and Zankov (1963b; 1968): ZPD (mental functions that require external stimulus e.g. "good teaching" that promotes the student's development) turns into ZAD (restructured mental functions that are used independently to solve certain tasks)



The individual ZPD of the child can be smaller or larger depending on the extent of which the child is able to utilize the collaboration in the classroom. The "objective" ZPD reflects mental functions needed to be formed during a given age period to form the next period (Chaiklin, 2003).

### Figure 3

Age, age period and different kinds of ZPDs in Vygorsky (apud Chaiklin, 2003)



Zankov writes that there are several ways in which teaching can affect the student's development. These pathways are not isolated but exist in a complex interaction. The creation of ZPD through teaching interacts with teaching's other ways of influencing the students' development of mental activity. Zankov also writes about "zones of proximate development" ("зоны ближайшего развития") (Zankov, 1963b, 16, author's translation) in which the paths of learning, which have a complex interrelation, can influence the student's mental development. If there are different "depths of the child's own mind" (Vygotsky, 1978, 45), there should also be different depths of the child's ZPDs. A deeper understanding of these processes, for which ZPD is a diagnostic principle, will lead to knowledge of the mutual relationship between learning and development (Zankov, 1963b). Gjære & Freiman (2022) write about a "zone of collective proximal development", meaning a symmetrical coconstruction of a ZPD and in a "creative polyphony" among the students. Zankov writes:

> The differentiated teaching methodology has a number of characteristics, the most important of which is that the same questions in the curriculum are worked through to different depths by different students. This allows the entire class, including the weakest students, to proceed ... (Zankov, 1968, 35-36, author's translation)

All students, with their individual mental "depths" and their individual ZPDs can contribute to the class's "average" or "larger" collective ZPD through collaboration with the teacher, differentiated teaching and creative "polyphonic" discussions and search activities together with other students and the teacher.

While the ZPD is often considered to include an asymmetric relationship between "a more knowledgeable other" and the student, a recent reworking of the concept has led to a more symmetrical model (Roth, 2020). The teacher's role can be dynamic where students guide the teacher towards their educational goals. The ZPD is not only dependent on the teacher, who selects appropriate learning activities, but also on the student's acceptance of participation in the activities. Breive (2020) has shown that the ZPD not only depends on the teacher, but also on the student's active contribution and commitment to achieving the learning goals together. In other words, the ZPD is viewed as a co-construction between the participants, where the teacher and the students almost form an "educational team" and where everyone is oriented towards the same goal for the activity (Radford, 2010). This is in line with Zankov's system's break with the "teacher-as-authority" culture that dominated the Tsarist system and, in the USSR (and elsewhere) during Zankov's time. A symmetrical perspective on the ZPD is described by Roth & Radford (2010). They begin with a critique of the ZPD:



Unfortunately, following a simplified reading of its original definition and primary sense in the quote that opens this text, the concept tends to be thought of in terms of the opposition of individuals. One of these individuals, a teacher or peer, is more capable than another individual, the learner (Roth & Radford, 2010, 299)

Based on this criticism, Roth and Radford developed a symmetrical perspective on the ZPD, i.e. "...in a reciprocal manner, the participants in a symmetric space of inter-action." (Roth & Radford, 2010, 304). They further write:

> The advantage of the symmetric approach to the zone of proximal development that we propose here is that it allows the question of the more capable subjectivity to emerge from the interaction, appropriate especially when the question who is in the know cannot be established on the basis of the institutional positions that the individuals otherwise take.

The ZPD is not only a "zone of agreements", but also a zone of "tensions, disagreements, misunderstandings, conflict, and subversion" (Roth & Radford, 2010, 306). But it is not connections between the didactic principles of Zankov that cause the tensions, but rather the level of difficulty, discussions, and the way of solving the problem. The ZPD reflects a certain "collision" between the teacher's and the student's perspectives. Zankov (1975/1977a) believes that such collisions can naturally emerge in the learning activities and thus create greater intensity. By overcoming these tensions, both the students and the teacher can learn and develop. This therefore takes place through a co-construction of the ZPD. A practice based on a sociocultural perspective can build on tensions between teacher and student to achieve increased student engagement (Brown, 2020). Zankov (1968, 1990) believed that one should allow students room to think, express doubts and questions and make mistakes, and considered mistakes as a stepping stone to a solution to the problem in a classroom with this kind of classroom atmosphere.

Zankov had the same definition and understanding of the ZPD as Vygotsky (Zankov, 1990, 1963b). Zankov talks about "child" and "adult" (Zankov, 1990, 94). The ZDP has an asymmetric perspective in the sense that the adult or the teacher or other children in primary education often is "the knowledgeable other" or "a more capable peer" in relation to the child. However, with Zankov, the teacher and the child are not in "opposition" (Roth & Radford, 2010) to each other. In this way, the ZPD has a symmetrical aspect in that the teacher listens attentively to the child, to impressions the child wants to share, impressions, doubts and questions that need to be answered (Zankov, 1968, 35). Here the teacher also gets the opportunity to learn and develop new approaches when it comes to didactics, pedagogy, and methodology. Zankov considered the nature of the relationship between the teacher and the student to be very important in his didactic system. An atmosphere of trust, cooperation and creativity is absolutely essential:

> The creation of a creative atmosphere and atmosphere of cooperation in the classroom is facilitated by the content of teaching and methods of work, which provide a variety of activities for students and allow each of them to fully participate in the educational process (Zankov, 1990, 19, author's translation)

"Vygotsky viewed the relationship he outlined between teaching and development as a working hypothesis and not as the solution to the problem" (Zankov, 1968, 45, author's translation). The ZPD can also be included in this working hypothesis. With Zankov's didactic system, the "solution" or the description of the relationship between teaching and development was closer than ever before. Focusing on how children develop long-term cultural concepts with "higher mental functions" rather than the short-term problem of helping children in their individual learning and problem-solving, was Vygotsky's primary concern. It is also wrong to connect the metaphor "scaffolding" to Vygotsky (Wood et al, 1976), as this has obscured Vygotsky's view of development together with the conflation of the ZPD concept (Smagorinsky, 2017; cf. Chaiklin, 2003).

According to Chaiklin (2003) Vygotsky used the ZPD term in his writings at least eight published texts. In general, the ZPD is mentioned in relation to development and learning in school age and to formal learning in school. The way to understand Vygotsky's ZPD concept is to understand the conceptual problem Vygotsky wished to address. Chaiklin (2003) sums up the Vygotsky's core understanding of this concept:

- Vygotsky distinguished between teaching that has the child's whole development and teaching technical skills. ZPD is therefore a concept related to development, not skills (generality aspect)
- More important for Vygotsky than the help that the child gets to do more and solve more difficult tasks so that he can become independent, is why this happens and why ZPD occurs (assistance aspect)
- Even if learning in ZPD not always is enjoyable, the child's potential is an indication of the existence of certain mental maturing functions, which in turn provide a potential for meaningful, interventional action (potential aspect). The ZPD was introduced by Vygotsky as a part of an analysis of child development. Its role is to indicate an important moment in child development
- The purpose of the ZPD is to identify the maturing mental functions that the child needs to transit from one age period to the next, and to identify the child's current state of developing these functions

- For Vygotsky the ZPD is a diagnostic principle that can help us understand the inner connections of the child's development process and the child's ability to imitate (within its intellectual potential) is the basis for the subjective ZPD, while the social situation in the child's development is the reason for the objective ZPD
- The ZPD of the child exists because of the maturing mental functions. The most important of the collaborative interventions in teaching is to indicate these maturing psychological functions. The interaction is used to assess the child's subjective ZPD because this gives an opportunity of imitation and thereby identifying the mental functions. Lower mental functions are genetically inherited, unmediated and involuntary, while the higher mental functions are socially acquired, mediated and voluntary controlled.
- The "larger" or "smaller" individual ZPDs of the children is determined by the child's ability to take advantage of collaboration beyond independent performance and age norms
- Vygotsky says that children with the same IQ can have different ZPD. A child can have a high IQ and a large ZPD or a small ZPD. Likewise, a child with a low IQ may have either a large ZPD or a small ZPD. Therefore, Vygotsky rejects IQ tests because they test family conditions rather than abilities. Optimal learning requires homogeneous groups and ZPD is suggested as an effective grouping principle (Barrs, 2022).
- The idea of ZPD is intended to direct teaching (obuchenie) attention to the maturing mental functions, instead of already existing functions. These functions are not created in interaction, but the interaction gives conditions for identifying their existence and how much they have developed.
- The ZPDs content and meaning will change from age period to another
- The focus on learning in school concepts in ZPD is due to the relevance of school age
- The function of the ZPD is to identify the development of the whole child
- The ZPD describes the relations between subjective maturing mental functions and objective functions needed for the next age period. This objective ZPD is the same for all children, but the individual ZPD of the child is positioned differently in the objective ZPD. The ZPD is not located in the child alone, but even the subjective ZPD. The subjective ZPD is an evaluation or assessment of the child's capabilities in connection to the theoretical age period

Chaiklin writes: "Vygotsky does not seem to have any systematic principles, methods or techniques that should guide how collaboration should be constructed by a person who is assessing a zone of proximal development" (Chaiklin 2003, 54). For such a system we had to wait for his student and colleague Zankov.

### Zankov: "Psychodidactics" and didactic principles

Leonid Vladimirovich Zankov (1901-1977) believed that there was a need for a new approach to didactics in primary school, with new principles and new didactics for a new and modern school. If the development of the children in primary school went very slowly and sluggishly, then it was necessary to construct a new educational system with new didactics (Zankov, 1963a; 1996). It was also necessary to arrange teaching and guidance in school "in such a way that it ignites in the child a spark which then flares up more and more - a thirst for knowledge ..." (Zankov, 1996, 4, author's translation). He conducted large-scale educational experiments to test Vygotsky's learning theory (Zankov, 1977a) in primary schools in the 1950s and 1960s, recognizing its fundamental importance. The experiments of Zankov represent a direct and continuous line from Vygotsky concerning educational practice. The Zankov's system sought to implement Vygotsky's views on pedagogy in large-scale and broad education. His experiments terminated in the 1970s, but his system saw a revival in 1996 when "Zankova" gained nationwide status and "was officially accepted as one of the three officially recognized variants of Russian elementary school education" (Boguslavsky, 2015, online). Asmolov uses the rather precise term "psychodidactics" for this direction, which creates a new didactic system that has the student's general mental development as its goal (Asmolov, 1998, 21), and where Zankov is the "undisputed leader of psychodidactics" (Asmolov, 2007, 15). Boguslavsky cites Asmolov: "Zankov's pedagogy is a pedagogy of interesting challenges and, while interesting challenges are life challenges, following Zankov means following vital didactics" (Boguslavsky, 2015, online). The relationship between teaching and development can be examined both as a problem of psychology and as one of pedagogy (Zankov, 1963b, 1968). The tradition of Vygotsky and Zankov regards this relationship as complex. Zankov says that in educational research psychological methodology and psychological analysis are needed (Zankov, 1968), because "psychology serves to reveal pedagogical patterns" and is a "support for pedagogy" (Zankov, 1990, 66, 104, author's translation).

Zankov's idea was to create a system of instruction and guidance in the primary (1-4) grades with a focus on the general (mental) development of children (Zankov, 1964). Zankov writes about the purpose of didactics and his didactic system:

> Since the development of students takes place in the teaching process, didactic principles and requirements aimed at acquiring knowledge and skills also produce certain results in terms of development. However, the task is not to achieve any

339

# iejee∽

[particular] result, but rather to achieve the maximum effectiveness of the lesson for the development of the students. However, the fulfillment of this task requires ... that the teaching process should be analyzed and structured from a special perspective (Zankov, 1968, 7, author's translation)

Zankov's idea is that didactics can no longer be reserved or limited to the area of knowledge and skills, no matter how important this is. It is necessary to develop scientific pedagogical foundations for a structure of the teaching process that ensures an optimal result in the development of students. It requires the development of new didactic principles, rules, requirements, and methodology – including curriculum, textbooks, and guidelines for teaching – to indicate the relationship between teaching and the students' mental development. While acquisition of knowledge, abilities and skills are the goal in the traditional system, they are just means for general development in the Zankov system.

The central idea of the didactic system is to achieve the highest possible efficiency in teaching for the overall development of the students (Zankov, 1968, 1977a). "By general development we mean development of the child's personality, all its sides" (Zankov, 1963b, 31, author's translation). That means the cognitive processes ("mind"), volitional qualities that govern all human activity ("will") and moral and ethical qualities manifested in all types of activity ("feelings"). Zankov transformed Vygotsky's theoretical ideas into a coherent and holistic system of curriculum and practice and tested it in public schools. Zankov formulated the main ideas of his new education system in five major principles (e.g. Zankov, 1968; 1973a, 1973b; 1977a, 1977b; Guseva & Sosnowsky, 1997). These five principles must always be understood as a unit; one of them cannot be removed from or underestimated in the system and they are interconnected on different levels. The problem of the correlation between learning and the development of the child's psyche was in the focus of Zankov's attention throughout all stages of his research activities. Zankov's major contribution was to link development's dependence on teaching through the study of development through a changed educational structure. The idea that a child has an inner world that is formed before school age, and that this inner world is a strong force that determines the direction of the child's mental development, runs like a red thread through the research and works of Zankov. In addition to the creation of the ZPD, another concept of Vygotsky is important for Zankov: the need to create conditions that ensure the unity of the functioning of affective and intellectual processes, which acts as an internal factor that determines development. The deep essence of Zankov's system is concretized in its didactic characteristics. The originality and pioneering spirit of Vygotsky and Zankov makes them deserve the title "harbingers of change" (Guseva & Sosnowski, 1997, 14). In Zankov, we not only have a learning

theory, but also, for the first time, a complete didacticmethodological system with concepts of activityoriented and personality-oriented approaches to teaching students in primary education and an interdisciplinary study, that combines pedagogy, didactics, psychology, paedology and teaching methodology. For the first time psychological methods were included as an integral part of pedagogical research. For the first time, the theoretical idea and the development of children became a practical implementation in the form of an integral system of education through an experiment. For the first time such a system organically combined pedagogical and psychological methods. A new educational goal had been formulated: to achieve the optimal overall development of each child. A new task had been put forward: a broad holistic world view by means of science, literature, art, and direct cognition. A new type of training had been created incorporating both a structure of training (principles, content, methods etc.) and attention on the inner world of the child.

The didactic principles have their own role, function, and nature in relationship to other principles (Zankov, 1977a). The mastery of knowledge is a means to an end. The implementation of the principles leads to "the birth, growth and deepening of an internal motivation for learning" (Zankov, 1977a, 62).

The didactic system is based not on some kind of isolated, heterogeneous principles, but rather on principles that are fundamentally interrelated, ... it is inconceivable for these principles to be realized separately ... (Zankov, 1977a, 61-62)

#### The leading role of theoretical knowledge

The didactic system emphasizes theoretical knowledge. Zankov writes:

The decisive role in elementary education should be played by the cognitive aspect, the theoretical knowledge, and not the cultivation of skills in language and mathematics. It is necessary to develop these skills on the basis of valuable general development, on the basis of deeper understanding of language regularity and mathematical connections and dependencies. (Zankov apud Guseva & Solomonovich, 2017, 779)

This principle, which includes the acquisition of scientific concepts, is about children being made aware of the characteristics of various phenomena and the connections between them. In the learning process in Zankov's system, students make different observations related to the material being studied. The teacher directs the students' attention and leads them forward towards finding the existing connections and dependencies in the studied material themselves. Students are guided towards explaining specific dependencies and drawing conclusions. The principle does not imply absolutization, nor does it prescribe to reduce the content of primary education programmes to scientific concepts. It requires that students, starting

from independent observations, constantly rise to the disclosure of interconnections and inter-dependencies between phenomena. The path from empirical cognition to abstraction and generalization should be made by the student (of course, at an accessible level) from the first steps of learning. This principle is important for the formation of the foundations of a scientific worldview in students. "Being able to use a scientific term presupposes the important condition of correct generalization and consequently the formation of concepts" (Zankov, 1973a, 41, author's translation). So, in the ZPD, the students' spontaneous concepts meet the teacher's theoretical concepts.

### Teaching at a high level of difficulty

One must teach at a so-called high (optimal) level of difficulty, "without making it too difficult" (Zankov, 1965, 19). Students try to overcome difficulties (challenging tasks and problems) in the zone of nearest/potential/ proximate development. Zankov writes:

Among the didactic principles of our experimental system a decisive role belongs to the principle of learning at the high level of difficulty. The concept of "difficulty" [TpydHOCTL, trudnost] is used in didactics in different contexts and in different meanings. One aspect of this concept – is "overcoming obstacles" (Zankov, 1990, 114, author's translation).

In elaborating this term, Zankov followed the works of the great Russian pedagogue Konstantin Ushinsky. His 19th-century monograph emphasizes the importance of overcoming the difficulties or obstacles to human development, which underlies this principle. Simply put: no obstacles, no development. The main idea of this principle is to create a special atmosphere in which the intellectual activity of school children is observed. Vygotsky is also clear on the necessity of difficulty in education:

> Thinking always arises out of difficulty ... it is necessary to take care to create as many difficulties as possible in the child's education, as starting points for his thoughts (Vygotsky, 1997, 173, 174)

In Zankov's system, "difficulty" is understood as the tension in the student's intellectual and spiritual forces, the intensity of mental work when solving problems, and overcoming obstacles that arise in the process of cognition. Sometimes this principle is misunderstood by claiming that the principle is inconsistent with nature. Such a characterization of the principle is then combined with an acknowledgment that education, if it is to be developmental, must be based on the child's ZPD. But one cannot have a ZPD without the principle of a high level of difficulty. The link between Vygotsky and Zankov is obvious when it comes to ZDP and difficulty: ... with collaboration, direction, or some kind of help [in ZDP] the child is always able to do more difficult tasks that [sic] he can independently (Vygotsky, 1987, 209)

"ZPD" is a psychological term with developmental-, educational-, and assessment-related aspects. In order to use it in the construction of education, it is necessary to translate it into a pedagogical plan; to fill it with pedagogical meaning. This is precisely what the aforementioned principle serves. It obliges one to look for the zone of proximate development of each student in the learning process; to search for the top step of the students' capabilities of a given class, which is not at all guided by the old principle of accessibility the idea that the education and upbringing of children should be age appropriate. The principle of teaching at a high level of difficulty is a prerequisite for creating the ZPD in the Zankov system. This principle of a high level of difficulty reflects, in a nutshell, the peculiarity of the psychological processes of the students when acquiring the subject matter. The effectiveness of this principle is due to the implementation of the educational material that is offered by the teacher and that can be comprehended by schoolchildren.

### Fast-paced progress

Zankov's method claims that a fast pace of study suits children's needs. They are interested in learning something new instead of repeating material that has been previously studied many times. Rather, they should repeat the material while learning something new. Vygotsky believed that children had to be taught something new and that the ZPD is an important "moment" for both education and teaching (Vygotsky, 1982). With Zankov's perspective:

> Making rapid progress in no way means undue haste in a lesson or hurried communication of the greatest possible amount of information to the pupils. Haste and the desire to set records are completely alien to our experimental system (Zankov, 1977a, 58)

Students' minds are better stimulated by variety than repetition and drill:

Repeating over and over to the pupils what they already know, resorting to reiterated monotonous repetitions, leads to mental indolence and to spiritual apathy, and consequently, hampers development (Zankov, 1965, 19)

This principle does not mean that in Zankov's system there is no return to the material already studied. The process of cognition is built in such a way that progress goes simultaneously with the return to what has been passed. Such repetition is aimed not at consolidating knowledge in the initially perceived form, but at a deeper comprehension of it. As a result, in the minds of schoolchildren, there are ever wider connections between the studied issue and others, which leads to the firm retention of knowledge in the students'



memory, and also contributes to the development of their thinking activity. When the development of the children is individual, so also must the high level of difficulty and rapid pace (organization and progression) vary from student to student, or at least among different student groups. Rapid pace or progress is linked to working in the ZPD since the children there will always learn something new; always pushing their limit and potential of mental activity, which differs for individual children (Vygotsky, 1982, 209). But it is a major mistake to think that this principle means great pace and haste in the classroom. The point is to secure progress, not to stress the children and risk their falling out of the ZPD. Essentially, this principle is based on Vygotsky: "The main function of education is to teach the child something new" (Vygotsky, 1982, 250) and is about enrichment of material and making the teaching continuously interesting and preventing it from producing boredom.

### Students' awareness of the learning process

With this principle, the method seeks to develop students' awareness of the learning process, a principle that is directed inward. The awareness and realization of one's own learning process – the dynamics of learning – are related to different mental activities. Zankov states:

> Thus, the process of mastering knowledge and skills itself becomes the object of awareness to a certain extent. How is the knowledge to be learned linked together? What are the various aspects of mastering spelling or computational operations? What is the mechanism underlying errors and the prevention of errors? These and many other questions relating to the mastery of knowledge and skills are the subject of unflagging attention on the part of school pupils (Zankov, 1977a, 60-61).

The principle of pupils' awareness of the learning process is very important in a system aimed at general mental development. This is a principle of educating schoolchildren in the ability to self-learn, to form their attitude to the world around them. The identification of the sprouts of reflection in schoolchildren, while studying their development, as the ability not only to realize the mechanism of the emergence of concepts, but also in a broader, personal sense, testifies to the effectiveness of this principle and its application by teachers. Formative assessment is central in connection with this principle, giving the opportunity to observe the process of their own learning. It relieves the feeling of being unnoticed by the teacher and confirms that the teacher cares about how the learning takes place. It shows when it is necessary to change the methods of work and adjust or vary the learning style. Finally, it helps students to become more effective, capable of self-organization and selfassessment. While the principle of consciousness is directed at the object of cognition, the principle of awareness of the learning process is turned inward to the student's own awareness of the process of learning.

# Planning of systematic development of each student in the classroom

A goal-oriented, systematic development of each student is sought by using the aforementioned four principles. This is also linked to the child's ZPD and applies to all children, including the "weakest" (Zankov, 1973b, 153). Zankov believes that this group of students has an even greater need than other students for systematic work to promote their development. Overloading these students with tasks causes them to lag behind in their development (Zankov, 1977a). He considers any isolation and division of students according to their progression, or creation of classes on the basis that students think alike, is contrary to the principle of development and to the very nature of teaching. All children should be able to move forward in their development. The development process takes place either slowly or unevenly, depending on the individual's potential, characteristics of the nervous system, personal experiences and so on.

Although Zankov's principles of theoretical knowledge, high level of difficulty and rapid progression can be perceived as the more "professional" principles, all five principles, including the principles of awareness and planning for all students' development, will form the whole of the teacher's work in the ZDP. The special development in individual subjects ("the direct path", Zankov, 1977a, 163) will contribute to a greater or lesser extent to the students' general mental development (the decisive role of "the indirect path", Zankov, 1977a, 163). Both paths are present in school teaching. Zankov believes that, in the right learning environment, each child can find their own path of development. The awakening of children's life in the classroom, the fully fledged process of their general development, is determined by the entire integrity of the system, including all its didactic components. But on the operational level, there can arise challenges regarding teaching in the ZPD, even with good learning resources. The principles of the experimental system are implemented in changing the content of primary education (curriculum), in methods of training and education, and in methodological construction work on academic subjects. "The difference between didactic systems is expressed in the nature of the relationship between direct and indirect ways" (Zankov, 1968, 127, author's translation). The system's methodology can illustrate this.

# Zankov: Methodological system with pedagogical properties

Zankov's didactic principles were created on the basis of findings from the practical teaching in his experiment, and these constituted a regulatory and directive role that was distinct from the daily teaching activity. Didactic principles are implemented through content, training and working methods. The teaching methods of the experiment determine the teacher's work.

> These teaching methods make it possible to realize the guiding idea of the experimental system and its didactic principles in the everyday activity of the teacher and in the pupil's learning (Zankov, 1977a, 274)

The common characteristic of these teaching methods is that they are based on the didactic principles. The didactic principles are on an abstract level and only the teaching methods communicate a didactic content to the teacher and the students (Zankov, 1977a). In this way, a methodologically uniform system is created with a specific structure and with typical pedagogical characteristics. These are revealed in the teacher's teaching methods and teaching techniques and in the students' study activities (observation activity, mental activity, and practical activity, Zankov, 1977a, 71-159), but primarily in the students' total mental activity.

#### Broad focus (versatility, multilateralism)

Broad focus means that teaching techniques have different functions on a methodological level. It is about learning resources (e.g. textbooks) that not only aim for the students to assimilate knowledge and skills, but also to promote the students' development. The teaching must also draw the pupils' actual broad mental activity into the learning sphere. It is a learning process that includes a diversity of students' personal experiences - emotional, intellectual, volitional, and aesthetic. This characteristic is distinguished by the richness of both the content and the form of the children's activity. Although one must develop knowledge, emphasis is also placed on developing emotions and a desire to learn. Pupils should have experiences that appeal to their emotional and aesthetic senses. Here we are talking about the joy of learning, sympathy, intellectual joy, but also dissatisfaction, disappointments, and doubts.

# The progressive nature of the teaching (process nature)

The procedural nature of the teaching methods operates at different levels in the educational process, in the communication of theoretical knowledge that reveals the essence of the didactic units studied, and the sequence of problems that the students encounter. The textbooks are not presented in the traditional sense, topic by topic. It is a continuous learning process where the students analyse and study individual elements, which can be assembled into a larger whole. Previous knowledge must be processed and viewed from new angles which, taken as a whole, will provide a more meaningful structure. In keeping with the process nature of our system of methods, each segment of a school course becomes a dependent element in a single, basic chain of components. (Zankov, 1977a, 276)

#### Confrontations (cognitive conflict)

In Zankov's didactic system, cognitive confrontations have a significant place. The nature of the confrontations reveals contradictions of full and incomplete knowledge, worldly and scientific ideas, and concepts. The confrontations support research activities and help develop skills such as reflection, and dialectical and critical thinking. Students often have misconceptions or facts that in some ways contradict what they will learn in class. By including these facts, one will, through conflicting information, set old learning against the new knowledge, which in turn will contribute to increased learning intensity among the students and to a deeper attainment of knowledge. This methodology can be linked to the principles of high difficulty, rapid progression, and theoretical knowledge.

> ... such [cognitive] confrontations should be used in every way to increase the intensiveness of the pupil's learning, to assure his ascent to every succeeding stage of learning activity and development (Zankov, 1977a, 277)

### Variation (flexibility)

The nature of the variation depends on different and specific conditions, especially the individual character of the pupils. Taking students' individual abilities into account can result in variations in the degree of difficulty of the material being studied. This includes variations in the degree of difficulty, rapid progression, task types, and didactic methods (individual work, group work, class discussion, etc.). Variation can also involve physical activity and "outdoor mathematics". Variation has an important place in the methodological system when it comes to the implementation of problems in practice.

> This property [variation] plays a functional role in the search for ways and means of altering the system of methods to accommodate permissible differences in the way the content of the educational process is structured as well as differences between teachers with regard to the style of work ... (Zankov, 1977a, 277)

The teaching methods provide direction for the individual characteristics of the various school subjects, for the degree of mastery in the various subjects, for how the students develop in the subjects, etc. The methods fulfil the didactic principles, but only after the didactic principles have contributed to crystallizing the methods in the various subjects. The methods take place, like the use of the didactic principles, in the ZPD, and again, as a comprehensive didactic and methodological system, contribute to the students' general (mental) development. It is the didactic principles that play a decisive role:



Boundaries on variation in the system of methods are set by the didactic principles. Variation - even significant variation - in the typical pedagogical properties of the system does not affect the operation of the experimental system as a whole, if the changes are in keeping with those didactic principles that characteristically play a directing and regulatory role (Zankov, 1977a, 277)

# Zankov: Different activities as indicators of general development

The general development of the student is the goal of Zankov's didactic system and the ZPD is a vital part of this system. But there are also certain activity traits that the system possess; that are indicators for achieving this general development: observational activities, mental activities and practical activities (Zankov, 1963b, 1977a). Zankov emphasizes that these activity traits must be understood as a whole:

> An analysis of general development that allows one to differentiate various lines within the process of development as a whole (observation, reasoning, practical activity) represent not a fractionation into elementary components but rather a separation into units (Zankov, 1977a, 67)

Zankov believed that students' development occurred according to three elements, as Vygotsky also presented it. We can talk about the students' development units: observation activities, mental activities and practical activities. When comparing experimental classes with traditional classes, Zancov asks:

> What typical features should a didactic system have, that will bring optimal results in the overall development of schoolchildren? (Zankov, 1990, 186, author's translation)

In Zankov's system, the overall idea is important. The didactic principles form a whole, in which no elements can be taken out or undermined, and the typical activity characteristics in the didactic system are also a whole.

### Observation activity

It is essential to study the development of observation since the process of perception is the basis for this. This activity is about perception, motivation, interest, and emotions. Perception of objects can differ in structure and complexity. This makes observation a complex activity. Perception is part of observation and is organically linked to reasoning. The observation activity incorporates a unique type of reasoning, and this reasoning process is directly based on sensory cognition of reality. Zankov states:

> There is no cause, however, to obliterate the significant differences between observation and reasoning. The object of observation is the object that is directly perceived. Cognition of this object consists in singling out, correlating, and combining

the data on its various parts, aspects, and properties by means of the senses (Zankov, 1977a, 71)

Observation brings a unique way of thinking. The thought processes are directly based on sensory cognition of reality and represent only the initial analysis and synthesis of sensory experience data (Zankov, 1963b), naming colours, shapes, and other properties; pointing out differences and similarities.

### Mental activity

Zankov's starting point is that mental activity is fundamental to the students' general development. Regarding the development of theoretical thought, Zankov refers to S. L. Rubinstein who explains when it develops:

> When theoretical reasoning develops, neither sensorimotor (visually active) nor visually figurative reasoning disappear, but are transformed and improved, and are themselves raised to the highest level. Quite diverse and complex interrelationships are created between them that vary individually from one case to another (Zankov, 1977a, 97; cf. Rubinstein, 2002, 449)

The basis for the study of the development of students' mental activity is the postulate of the diversity of thought forms, their substantial mutual relations, and their inter-conditioned movement. The qualitative changes in thought occur in an upward movement, from the lower mental functions (reactive attention, sensation, sensorimotor intelligence, and associative memory) to the higher mental functions (mediated perception, focused attention, deliberate memory and logical thinking, including analysis and synthesis, abstraction and generalization, which are socially created, voluntarily controlled, semiotically mediated and united in systems with other functions). For Zankov, this means that the concept is developed, expanded, and deepened in the subject's subsequent approximations to the object. This movement was the focus of Zankov's investigation in connection with mental activities, and the quality of the objects of knowledge is the basis of the observation, not the word, that Vygotsky focused on. In this movement of abstraction and generalization, of analysis and synthesis, from the first forms of perception of the ordinary character of the objects, there is a transformation of the object being studied into a "new" object. Each time this movement is repeated, and the process is carried out, new characteristics are found. This is the reason behind the postulate that the study of mental activity of students is the diversity of thought forms.

In his didactic system, Zankov used algebraic symbols for generalization. By using such methods, the system achieves the students' in-depth understanding of relevant connections and dependencies. The mental activity will affect the students' ability to reach a higher level of abstraction and generalization (Zankov, 1968).

### Practical activities

Zankov (1977a) states that the development of practical activity constitutes a significant part of a child's general development. Practical object-based activities involve more than pure motor skills and habits. It also reflects the sense sphere, spatial ideas, and reasoning. Overcoming difficulties associated with practical activity also reveals certain emotional and volitional aspects of psychological activity. "A specific characteristic of practical activity is the fact that a unique relationship exists here between thought and action" (Zankov, 1977a, 128). There is thus no distinction between mental activity and practical activity. One cannot be performed without the other. They have their peculiarities, but they are equally a unit. In the practical activities, Zankov is interested in analyzing the means and action procedure in the task, but also the qualitative aspects of the action, speed, precision, self-control, and the nature of the mistakes.

The phases of the students' development activities, observation activity, mental activity, and practical activity are treated as units because one is present in the other. These activities also make clear the emergence from the abstract to the concrete, a procedure that is important for the materialist didactic method. These three development activities form an important unit in Zankov's didactic system. The didactic principles are closely related to properties of the methodological system; in essence, means of implementing the didactic principles. Discovering the true connection between learning and development requires a more accurate determination of the kind of knowledge that is learned and how it is learned by the students.

### Russian mathematics in Norwegian classrooms: Challenges

Zankov's didactic system has elements in it that can potentially make significant contributions to school children's development, as it has clear and justified relationships between teaching and the students' development. The "miracle" that the teachers in Zankov's experiments talked about

> ... consists in the fact that our experimental education awakens and develops the children's creative powers and abilities (Zankov, 1964, 6)

This is in line with Vygotsky: "... learning awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers" (Vygotsky, 1978, 90). This means that

an essential feature of learning is that it creates a zone of proximate development. Zankov says that it is impossible to achieve results, especially in teaching and development, if the teacher does not know the individual student's character and psyche (Zankov, 1964, 4). Among other things this "miracle" and "awakening" is what makes Zankov's system attractive.

Zankov's system is not used in all subjects at primary school level as it was originally developed for. However, Zankov's principles and methods have been used in Norway for the subject of mathematics in particular. Although the focus is perhaps more on learning than development in the education of Norwegian pupils, Zankov's method has nevertheless proven to be potentially effective for teaching and learning mathematical competence. Consequently, this should not be the reason for challenges in implementing the Zankov method in mathematics. What about potential cultural differences between Russian and Norwegian pupils? This could be a plausible explanation, but here we seek explanations for implementation problems regarding Norwegian teachers because of the central role of the teacher in this system. The teacher is the planner, facilitator and mastermind behind the introduction and implementation of the Zankov system.

Lieberg (2015) warns that "Russian mathematics" is not "magic dust" in mathematics education in Norway. He emphasizes that the method is demanding and requires guidance and professional support to teachers who are going to use this method. No adequate research has been carried out in Norway about the teachers' implementation of the system nor the students' results linked to Zankov's system.

Gjære (2022) points out some potential problems with this very implementation. He refers to the observations of a teacher who does not challenge the pupils to work in the ZPD. Part of the problem is that the problems worked on are too simple and that the students are not challenged enough and therefore do not follow the didactic principle of optimal difficulty level in the classroom. This teacher is working in "yesterday's development" according to Vygotsky and Zankov (Zankov, 1990, 12, 94). There is not necessarily a problem with the teacher's loyalty to Zankov's model, but it can still be challenging to implement DEM and thus the ZPD in the classroom.

Other challenges that Gjære (2023) raises in connection with the implementation of DEM is linked to the term "teaching dilemmas" (Lampert, 1985, 181), a concept of Lampert's concerning childrens' attention. Gjære (2023, 5) addresses the following three "dilemmas" that he encountered with some teachers who teach mathematics according to Zankov's model:



- the dilemma of telling (the teacher is telling the students the answer too quickly and showing them methods when there is a lack of response from the students);
- the dilemma of bored students with important tasks (the teacher is sticking to the same discussion when the students are ready to move on);
- the dilemma of keeping pace (the teacher must give all the students appropriate challenges at different levels while the learning work in the class must be continued).

A teacher's workday is full of challenges and choices, including for "the Zankov teachers". Gjære hints that teachers can be controlled too much by the guidance material in order to teach in "the right way" (Gjære, 2023, 6). They can also let themselves be stressed by the demands for teaching with a high level of difficulty and rapid progression. Although these were important and central didactic principles in Zankov's system, there is reason to recall that Zankov himself said that the teacher must teach at a so-called optimal level of difficulty "without making it too difficult" (Zankov, 1965, 19). Furthermore, he also said that one should not be hasty in the classroom:

> Moving forward at a fast pace does not mean rushing through the lesson ... The fast pace of learning makes it possible to reveal different aspects of the acquired knowledge, deepen it and connect it (Zankov, 1990, 117, author's translation)

Equally unacceptable are "monotonous repetitions" (Zankov, 1965, 19). A "dilemma" can appear for the teacher: By following the teacher's planning too strictly about, for example, time use, one can end up breaking with Zankov's didactic principles. It is important that the teacher has an overall view of DEM and tries to follow the principles in their daily teaching. The teacher's planning and didactic reflections on the teaching are crucial for success with Zankov's model. The implementation of DEM according to Zankov's didactic principles in Norwegian classrooms depends on the teachers having the necessary training, the required flexibility and creativity, and adequate reflection on Zankov's system and the ZPD concept. Examples of threats to Zankov's system are too high or low a level of difficulty, or a lack of flexibility and patience on the part of the teacher. The challenge of teaching in the ZPD according to Zankov's didactic system should be addressed. Successful implementation of the Vygotskyan and Zankovian ZPD concept is a prerequisite for a successful implementation of Zankov's didactic system. Deep knowledge about the ZDP and Zankov's didactic principles is crucially important for a successful implementation.

Gjære (2023) writes about "dilemmas" that teachers experience during the Zankov DEM lessons and says

that "the origins of these dilemmas were found to lie in internal tensions between the components of the DEM system" (Gjære, 2023, 1). Zankov says that the teachers must look at the process of teaching with new eyes. He never mentions "dilemmas" in connection to working with the principles in the ZPD in the classroom or "tensions between various principles" (Gjære, 2023, 8). Going back to the early days of Zankov's experiments, there were "slips and mistakes" and it was not easy for the teachers to shift from former methods to the new form of instruction, "But all the trouble is well compensated" (Zankov, 1964, 6). The teachers may well be experiencing "dilemmas" and challenges, but rather than real conflicting dilemmas of instructions inside of Zankov's didactic system and in the ZDP, these "dilemmas" are more likely to occur when the principles and methods are not followed or are misunderstood. For instance, the principle about fast pace is not about "rushing through class" (Zankov, 1968, 34, author's translation), but more about continuous enrichment, learning something new and even deeper understanding. There are also no contradictions between the principles of high difficulty level and fast pace. On the contrary: a high level of difficulty presupposes a fast pace (progression) and they are "organically connected" (Zankov, 1968, 34, author's translation). Furthermore, the didactic principles constitute a whole, an integrated system, wherein the principles are interconnected. Keeping the pace and planning for development for all students should therefore not be perceived as a dilemma. To reiterate, fast pace does not mean haste or stress in the classroom. This principle was a reaction to the traditional school's "slow pace" resulting in "chewing gum", monotonous repetitions, mental laziness and spiritual apathy, leading to hampering the children's development. In the Zankov system, this principle is easy to misunderstand because of its very name. The principle of theoretical knowledge, in combination with a high level of difficulty and fast paced progress, could lead to an unjustified increase in intellectual load. These kinds of misunderstandings of Zankov's system could contribute to violating the conceptual ideas of the system. In the Zankov system the importance is not only placed upon the principles, but also on how those principles are observed in the teaching methodology. It is therefore likely, or possible, that the implementation problems could also be due to other potentially deeper problems, e.g.:

- lack of adjustment to the new system (a halfway implementation would not do) and being stuck in old didactic methods and thinking;
- lack of deep knowledge of Zankov's didactic system, the didactic principles, the properties of the teaching methods and the indicators for mental development;
- misunderstandings around Zankov's system, thinking that the didactic principles are in

conflict and result in "dilemmas" / do not constitute a whole;

- lack of reflection and planning around the Zankov lesson;
- lack of reflection about the general purpose of Zankov's system;
- lack of professional and didactic knowledge;
- lack of knowledge about children's psychology (pedagogical psychology);
- lack of "pedagogical patience" (tactfulness and sensitivity towards the learner's activity) and the ability to "sacrifice the moment for the future" (F. Schleiermacher, apud Mielityinen-Pachmann & Uljens, 2023, 207); when mental functions are maturing during the work in overcoming problems
- lack of pedagogical and didactic flexibility and creativity (e.g. too attached to textbooks and teacher's guides in relation to the planned lesson);
- lack of motivation because the decision to introduce the model does not lie with the teacher themself but, for example, at a municipal level;
- lack of knowledge of the problem-solving method (cf. Polya, 2009)
- possible cultural differences

The proposed teacher dilemmas in Gjære (2023) most likely occur because the teachers are unconsciously violating Zankov's principles for working inside the ZPD. This violation may well be explained among, or from, the abovementioned deeper problems. The Zankov system does not have built-in dilemmas, but it heavily depends on the teachers' ability to implement the system – a system of elements that is not externally united with each other, but:

> [The didactic] system is a whole that represents a unity ... An essential feature of a system is its integrity. A system is characterized by the interdependence of its parts, and connections between them (Zankov, 1968, 86, author's translation)

The Zankov system with its principles and methodology is characterized by integrity, interdependence, and inter-connectivity – not tensions, dilemmas, or contradictions. The implementation of Zankov's system will hardly be perfect and slips and mistakes will occur. What is alarming is if these slips and mistakes are introduced into a new "dilemmic" system. Instead of "telling", there should be guiding, leading questions, hints, discussions, flexible adjustment of the problem, including students' suggestions and questions, etc. Instead of "bored students" (a threat to development according to Zankov), the teacher should have fasterpaced progress or a selection of more difficult and challenging problems; spontaneously expand the

complexity around the same task; or assess whether it is appropriate to discuss a task in plenary or not. The dilemma of "keeping pace" should be adjusted by giving necessary time for everybody if a problem is presented for the whole class, or selecting fewer problems (Zankov, 1977a; 1990). There should be no rush or haste in the Zankov classroom. An important point about progression (fast pace) is that the students do not get bored; rather it includes variation and enrichment. The idea is maximum learning efficiency for general development of schoolchildren, and the goal is to construct teaching for this development (Zankov, 1968). Zankov states: "This principle [high level of difficulty] is closely related to a fast pace of material flow" and "... general development requires a certain complication of the educational material" as well as "Masticating what is learned is harmful to development" (Zankov, 1963b, 40, author's translation). No dilemma. No tension. Only organically connected and related principles - meaning: "... continuous enrichment of the children with ever-new knowledge ... [and] ... refusal ... from monotonous repetition of the past" because "Moving forward at a fast pace does not at all mean rushing through class" (cf. Zankov, 1968, 34, author's translation). Because in an effective learning system you can ask different questions and use "differentiated methods" within a topic for different students and to a different depth. "Thanks to this, the whole class, including the weakest, can move forward quickly" (Zankov, 1968, 35-36, author's translation). Therefore, the planning for every student's development and fast pace does not constitute a dilemma between these two didactic principles in the Zankov system. The principle of theoretical knowledge requires that the cognitive domain of teaching comes to the fore, both as a powerful means of student development and as a reliable basis for the acquisition of skills. Therefore, the principle of theoretical knowledge (interrelated phenomena, terms. logic, abstraction and generalization) and planning for students' development are also connected principles. Likewise, "... the connection between learning at a high level of difficulty with other 'principles' is expressed in the fact that there is ... the difficulty that finds its expression in mastering theoretical information, or a difficulty that promotes awareness of the learning process by schoolchildren" (Zankov, 1968, 87, author's translation). Zankov writes, for instance, about "difficulty" as "complexity of the educational material" and simultaneously as the "tension of strength" [напряжение сил] of the students, but never about tensions between didactic principles. I can therefore not agree that the Zankov system contains tensions and dilemmas between various didactic principles. Implementation problems must probably be explained by other factors.

Lack of flexibility can be an important reason for "dilemmas" arising in the classroom. "Flexibility in



organizing the teaching ... is absolutely essential ..." (Zankov, 1977a, 38) and is an important property or characteristic of the Zankov system: "The problemsolving experience ... is flexible ..." (Zankov, 1977a, 202). The teacher must regulate the help and the progression along with the level of difficulty to avoid these "dilemmas". If not, this could be a sign of not working in the ZPD and thereby not implementing Zankov's didactic system.

> Teachers differ from one another in many ways - cast of mind, character, work style, experience, professional qualifications, and so on. Since the implementation of our experimental didactic system requires a radical reorganization of the teacher's work, flexibility in altering one's general pedagogical approach to teaching and upbringing becomes especially important. Teachers possess this quality to varying degrees. The task consists in identifying the necessary and essential features and studying them as they combine with individual characteristics in the practical work of the teacher who is implementing the experimental didactic system (Zankov, 1977a, 34)

Some words should also be said about the method of problem-solving in Zankov's experimental system: "Problem solving plays a large part in experiential learning" (Zankov, 1990, 132, author's translation). Zankov refers to G. Polya and his publications (Zankov, 1977a, 183) in which a significant place is given to reasoning processes in the problem-solving. Zankov singled out four features of a problem: the conditions, the question, the data and the unknown. Zankov writes:

> To work successfully with problem text, the pupil must have a clear understanding of the problem, of its various parts, and of the kind of information contained in each part. He must be able to find the conditions of and question in the problems, regardless of how they are arranged in the text or the form in which they appear. He must also be able to find data in the text, an must understand what he need to look for (Zankov, 1977a, 182)

An important part of Zankov's methodological system is therefore the problem-solving method. It is quite significant and essential that the Zankov teachers have a solid knowledge of this method when they seek to implement Zankov's system in the classroom, to give the students solid instruction of what a "problem" is and "how to solve it" as Polya writes (Polya, 2009). A teacher's work with the "Car Problem" referred to by Gjære (2023, 7-8) may indicate lack of knowledge of the problem-solving method, not only lack necessary flexibility. In the Zankov system the principles of high level of difficulty (the problem) and the application of theoretical knowledge (concepts) of the problem, and the problem-solving (the method of reasoning, including search activity) is connected. All the didactic principles are included in this method, and failing here is to fail the implementation of Zankov's system at its heart.

When teachers do not succeed in implementing developmental education in mathematics (DEM) according to Zankov's system and are not working in the students' ZPD, we should look for explanations in the teacher's lack of knowledge of Zankov's system (didactic principles, methodology and characteristics of mental development), but also in the teacher's general pedagogical approach to learning and development. Asmolov (2012) claims that the primary school teacher is the master of navigation of the general development of the child. Furthermore, he (Asmolov, 2012, 6) refers to a saying in Russia: "3a системой Занкова будущее" ("the future belongs to the Zankov system"). The future will show whether Norwegian primary school teachers are capable of implementing the Zankov system. Zankov is clear when it comes to the implementation of the didactic principles:

> Our didactic principles are interconnected. However, these connections are not uniform: they operate on different levels and differ in their role and function. The principles of the experimental system are implemented in the construction of the content of primary education, in the methodological construction of work on academic subjects. The experimental system covers all primary education as a whole, and not individual academic subjects or parts thereof. The basis of the system is not an isolated, heterogeneous provision, but principles that are organically connected with each other (Zankov, 1990, 119, author's translation)

It is like instruments that have their own role but constitute a whole harmonic orchestra. "Every principle is manifested concretely, in keeping with its role in the didactic system, its functions, and the nature of its relationship to other principles" (Zankov, 1977, 62). Just as teaching and development is "organically connected" [органически связанные], so are the didactic principles, principles that form a cohesive whole and involve processes or connections that occur naturally and have a common goal. The didactic principles cannot be implemented separately from other principles. "Each principle is specifically revealed in accordance with its role in the didactic system, its functions, as well as features of its connections with other principles" (Zankov, 1990, 119, author's translation). Importantly, "the implementation of our didactic principles leads to the emergence, growth and deepening of the internal motivation to learn" (Zankov, ibid.). "Thanks to the teaching methodology, the guiding idea of the experimental system and its didactic principles are implemented in the daily activities of the teacher and the teaching of schoolchildren" (Zankov, 1990, 113, author's translation). Lampert's terms "conceptual paradoxes", "teaching dilemmas" and "contradictory aims" (Lampert, 1985, 181) do not apply to Zankov's didactic and methodological system. Instead, the Zankov system is characterized by integrity, uniformity (not in dilemmas) of goal and interconnectivity where the didactic principles are concerned.

Today, there is relatively little international research published in English on the Zankov method. In Russia, the method has been well established since 1995/1996 alongside the Elkonin-Davydov method and the "traditional" method. The Zankov model has had some spread beyond Russia's borders, such as in Norway (since 2009) and Iceland (since 2017). How does a "Russian" education system that was developed in the Soviet Union in 1957 - 1977 function in 21st century postmodern Norway and a different cultural context? The first DEM project in Norway in 2009 seems to have been successful (Gjære & Blank, 2019), but what about the other teachers who followed and used the same method? Some experiences in the use of the Zankov method have been uncovered (Gjære & Blank, 2019; Gjære 2022, 2023). The method is demanding and there are minor and major errors in the implementation of the method. This also happened when Russian teachers tried to implement Zankov's method in the Soviet Union for the first time. It was not easy to change from the old "traditional" methods to the new. "Slips and mistakes" were made, but the problems were "compensated" (Zankov, 1964, 6). It is important to avoid these "slips and mistakes" becoming systematic within the didactic system. The Zankov method offers teachers in primary education a whole system of didactics and methodology as an alternative to Norwegian mathematics teaching, which is often characterized by "traditional ways, focusing on routine skills, memorization of isolated facts and algorithms, relying on textbooks [including explanations and answers and solutions]" (Gjære & Blank, 2019). To implement the didactic system, the teachers need to work systematically and understand the inter-connectivity among the principles. Small "slips and mistakes" by Norwegian "Zankov teachers" in some situations are not certain to destroy their whole implementation of the Zankov system. The implementation depends on whether these "slips and mistakes" are systematically executed inside the didactic-methodological system or are symptoms of more fundamental challenges. A possible, and not unreasonable, explanation for the lack of implementation could be that the teachers do not familiarize themselves with the method well enough (cf. Gjære 2022 and his proposal about "depth" in the implementation of DEM). In that case, this is a very poor starting point for a successful implementation. Another reason could be that the method is regarded as a pedagogical "quick fix" among other teaching methods and strategies that invite "high hopes" but end with "a shot in the dark". Another possible reason could be the teachers' lack of belief in their own students and their ability to learn more than we traditionally expect - stuck in well-trodden didactic and methodological paths and attitudes.

We should be concerned about the long-term effect of teachers' practice in the classroom. This should also be the focus point in future research. We should be able to follow teachers from grade 1-4 using classroom observations, interviews, and questionnaires (selfreporting) over a four-year period to uncover how the teachers work to implement this method. After this period, we should be able to survey the students' results on the national test at the beginning of grade 5. The first DEM project in Norway indicates that the Zankov method has great potential to lift the students' competence and learning outcomes and broaden their horizon in mathematics, constituting an interesting didactic and methodological alternative for Norwegian primary education teachers that can guide low performing students to a higher level on "the ladder of difficulty" and promoting every student's potential and development. To uncover and to utilize this potential, a systematic long-term research project is needed

A critical factor for teachers' successful practical implementation of the didactic-methodological system of Zankov is a deep knowledge of both the general and specific features of the system. Zankov's system is not primarily a system for learning per se in mathematics, language or other subjects. But through Zankov's development of new specific didactic principles, content, and methodology (characterized by integrity, consistency and interconnectivity in a system as a whole), these subjects can contribute to the optimal effectiveness of the (educational) development of the children. All the children's learning processes include not only the rational, but also the emotional sphere. The children learn new things in a continuous enrichment of their minds because they have an inner drive and motivation to learn. Every lesson is to be interesting and characterized by both a positive attitude, difficulty, and the joy of learning never boring and monotonous. Moreover, the children should always be aware of why they are learning. All the children are being challenged, and the goal is individual development for all. Even if there is a system of didactic principles, content, and methods to follow, the teacher must never forget flexibility among many other professional, pedagogical, and didactic properties. The key provision of this system is that it is built on the ideas of developmental variable education and the system-activity approach. Zankov put variability at the forefront of his system. Variability for him was expressed in the flexibility of the learning process. But this variability and flexibility does not apply to the didactic principles causing a flawed understanding of his system and the consequent difficulty in implementing his system. According to Zankov's system of developmental education, any child may be able to learn and develop, but not every teacher is ready to teach according to it. Those teachers who want to have a real pedagogical, didactic, and methodological challenge to overcome and master, must work cleverly and creatively and



flexible, not rigidly. Zankov's system provides the teacher with a didactic theory and methodology for the development of the childen's personalities and their cognitive and creative abilities. Zankov's system has great potential for the formation of motivation for learning and cognition, but the system's challenge is equally great.

### Footnotes

"Proximate" ("nearest", "next") rather than the traditional use of "proximal" is probably closer to the Russian original of Vygotsky's work, (cf. Vygotsky, 2017, 371, note 4). зона ближайшего развития (ЗБР), zona blizhayshego razvitiya (ZBR). ближайшего (blizhayshego), although it could mean "close", "near", "intimate", "nearby", "imminent". "Proximal" in the English phrase is a rather obscure biological term, first used in Mind in Society (Vygotsky, 1978). I use "proximate" except for some citations from other authors and translations. Cf. Barrs (2022, xvii).

### References

- Asmolov, A. G. (1998). Vygotsky Today: On the Verge of Non-Classical Psychology. A Volume in Horizons in Psychology. Series Editor: James Wertsch, Nova Science Publishers, Inc., New York
- Asmolov, A. G. (2007). Teoriya Zankova i sovremennoye obrazovaniye [Zankov's Theory and Modern Education], *PRAKTIKA OBRAZOVANIYA*, 3/2007, 14-17
- Asmolov, A. G. (2012). Sovremennaya shkola: razvivayushchaya i razvivayushchayasya [Modern school: developing and evolving], № 2 (58) OBRAZOVATEL'NAYA POLITIKA, 2-6
- Barrs, M. (2022). Vygotsky The Teacher. A Companion to his Psychology for Teachers and other Practitioner, Routledge. Taylor & Francis Group. London and New York
- Boguslavsky, M. V. (2015). Vital Pedagogy of Leonid Zankov, Russian-American Education Forum: An Online Journal, vol 7, issue 3. www.informaticsalliance.com/content/ en/?task=art&article=1001153&iid=23
- Breive, S. (2020). Student-teacher dialectic in the co-creation of a zone of proximal development: an example from kindergarten mathematics, *European Early Childhood Education Research Journal*, 28:3, 413-423, DOI: 10.1080/1350293X.2020.1755498
- Brown, R. (2020) Re-conceptualizing the development of agency in the school mathematics classroom, *Theory Into Practice*, 59:2, 139-149, DOI: 10.1080/00405841.2019.1702394

- Chaiklin, S. (2003). The zone of proximal development in Vygotsky's analysis of learning and instructions, in Kozulin, A., Gindis, B., Ageyev, V., Miller, S., Vygotsky's educational theory and practice in cultural context. Cambridge University Press
- Gjære, Å. L. & Blank, N. (2019). Teaching Mathematics Developmentally. For the Learning of Mathematics, November 2019, Vol. 39, No. 3 (November 2019), pp. 28-33, FLM Publishing Association
- Gjære, Å., Freiman, V., (2022). Expanding a Collective Zone of Proximal Development. Investigating the Potential of Zankov's Lesson with Elementary Students Analyzing Numerical Equalities, in Mathematics as the Science of Patterns: Making the Invisible Visible to Students Through Teaching, 151-182, Information Age Publishing
- Gjære, Å. L. (2022). Depth as a key issue for implementing DEM: The case of a teacher. I: Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education (CERME12). European Society for Research in Mathematics Education, 1-8.
- Gjære, Å. (2023). Challenges of Teaching with Challenging Tasks: Teaching Dilemmas Arising FromImplementingaReform-orientedApproach to Primary Mathematics, *Mathematics Teacher* Education and Development, vol.25(2), article 2
- Guseva L. G. & Sosnowski, A. (1997). Russian Education in Transition: Trends at the Primary Level, *Êducation canadienne et internationale* vol. 26 no. 1, 14-31
- Guseva L. G. & Solomonovich, M. (2017). Implementing the Zone of proximate Development: From the Pedagogical Experiment to the Developmental Education System of Leonid Zankov, International Electronic Journal of Elementary Education, 9(4), 775-786
- Lampert, M. (1985). How do teachers manage to teach? Perspectives on problems in practice. *Harvard Educational Review*, 55(2), 178–195. https://doi. org/10.17763/haer.55.2.56142234616x4352
- Lieberg, S. (2015). Russian mathematics is no magic dust ["tryllestøv"] in mathematics education (author's translation). Hadeland (Newspaper). Readers letter. December 18
- Longarezi, A. M. & Ferola, B. C. (2023). Desenvolvimento de estudantes no processo de aprendizagem, *Obutchénie: R. de Didat. e Psic. Pedag.*|Uberlândia, MG|v.7|n.2|1-29|mai./ago. 2023, DOI: http://doi.org/10.14393/OBv7n2.a2023-70265

- Mielityinen-Pachmann, M. & Uljens, M. (2023). Hermeneutics in the Non-affirmative Theory of Education in Non-affirmative Theory of Education and Bildung (chapter 9). DOI: 10.1007/978-3-031-30551-1\_9
- Newman, F. & Holzman, L. (1993). Lev Vygotsky: Revolutionary scientist. London: Routledge
- Polya, G. (2009). How to Solve It? A New Aspect of Mathematical Method, Ishi Press International
- Radford, L., (2010). The Antropological turn in Mathematics Education and its implication on the meaning of mathematical activity and classroom practice. *Acta Didactica Universitatis Comenianae Mathematics*, 10, 103-120.
- Roth, W. M. (2020). Zone of Proximal Development in Mathematics Education, *Encyclopedia of Mathematics Education*, 913-916, Springer
- Roth W. M. & Radford L. (2010). Re/thinking the zone of proximal devolpment (symmetrically). *Mind, Culture, Activity* 17(4), 299-307
- Rubinstein, S. L. (2002). Osnovy obshchey psikhologii [Fundamentals of General Psychology], Series: Masters of Psychology, Peter
- Smagorinsky, P. (2017). Deconflating the ZPD and instructional scaffolding: Retranslating and reconceiving the zone of proximal development as the zone of next development, *Learning*, *Culture and Social Interaction* 16 (2018) 70–75.
- Toulmin, S. (1978). The Mozart of psychology. New York Review of Books, 25, 51-57, September 28
- Vygotsky, L. S. (1978). Mind in society. The development of higher psychological processes. Cambridge, MA: Harvard University Press
- Vygotsky, L. S. (1982). Thought and speech. In Vygotsky, L.S. The collected works in 6 volumes. Vol. 2. Moscow: Pedagogika (in Russian). First published (1934).
- Vygotsky, L. S. (1987). *Thinking and speech* (N. Minick, trans.). In R. W. Rieber & A. S. Carton (eds.), The collected works of L. S. Vygotsky: Vol. 1. Problems of general psychology (pp. 39-285), New York: Plenum Press. (Original work published in 1934)
- Vygotsky, L. S. (1997). Educational Psychology, St. Lucie Press; Pedagogicheskaia psikhologiia 1991 Moscow Pedagogika [Pedagogical psychology, 2nd ed.]

- Vygotsky, L. S. (2001). Tenkning og tale [Thinking and speech]. Oslo: Gyldendal akademisk (Norwegian)
- Vygotsky, L. S. (2017). Problema obuchenija i umstvennogo razvitija v shkol'nom vozraste [The problem of learning and mental development at school age], *Changing English*, 24:4, 359-371, DOI: 10.1080/1358684X.2017.1395165
- Wood, D., Bruner, J. D., & Ross, G. (1976). The role of tutoring in problem solving. Journal of Child Psychology and Psychiatry, 17, 89–100. Retrieved from https://isites.harvard.edu/fs/docs/icb. topic862383.files/Wood1976.pdf
- Zankov, L. V. (1963a). The didactic fundations of Teaching (Submitted for Discussion), *Soviet Education*, 5:4, 3-12
- Zankov, L. V. (ed.). (1963b). Razvitiye uchashchikhsya v protsesse obucheniya (I — II klassy) [Development of students in the learning process]. Moscow: Academy of Pedagogical Sciences of RSFSR (in Russian). Summary in CEAS Abstract Series No. 1-4 (ED 060 227)
- Zankov, L. V. (1964). Convincing experience, Soviet Education, 6:3, 3-6, DOI: 10.2753/RES1060-939306033
- Zankov, L. V. (1965). Experiment with a New System of Primary Education, *Soviet Education*, 7:3, 17-29 https://doi.org/10.2753/RES1060-9393070317
- Zankov, L. V. (1968). Didaktika i zhizn' [Didactics and life]. Moscow: Pedagogika (in Russian).
- Zankov, L. V. (1973a). Didaktik und Leben [Didactics and Life]. Hannover: Schroedel (in German). Summary in https://files.eric.ed.gov/fulltext/ ED133180.pdf
- Zankov, L. V. (1973b). The Contribution of Studies in Experimental Psychology to Pedagogical Research, Soviet Education, 15:5-6, 146-176
- Zankov, L. V. (ed.). (1975). Teaching and development ['Obuchenie i razvitie']. Moscow: Pedagogika.
- Zankov, L. V. (1977a). Teaching and development: A Soviet Investigation, White Plains, NY: M.E. Sharpe.
- Zankov, L. V. (1977b). Principles of the experimental didactic system. *Soviet Education*, 19(4-6), 52-64. DOI: 10.2753/RES1060-939319040652

# iejee∽

- Zankov, L. V. (1990). Izbrannyye pedagogicheskiye trudy [The selected pedagogical works]. Moscow: Dom Pedagogiki.
- Zankov, L. V. (1996). On the issue of correlation training and development! From a report at a teachers' seminar in Moscow in June 1964. Published from the book: Commonwealth of Scientists and Teachers, Psychological Science and Education 1996, No. 4 (24-28). Russian
- Zaretsky, V. K. (2016), Lev Vygotsky's Principle "One Step in Learning Represents a Hundred Steps in Development": From Idea to Practice. Revue internationale du CRORES: innover dans la tradition de Vygotsky (December 2016) ISSN: 2291-6717, vol 4, no 1, 97-133
- Zaretsky V. K. (2021). One More Time on the Zone of proximate Development. *Cultural-Historical Psychology*, Vol. 17, no. 2, pp. 37—49. DOI: https:// doi.org/10.17759/chp.2021170204