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Open-ended Tasks in the Promotion of Classroom Communication in Mathematics

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Abstract

Mathematics programmes in basic education are currently undergoing reform in Portugal. This paper sets out to see how teachers are putting the new guidelines for the teaching of mathematics into practice, with particular emphasis on maths communication in the classroom. To achieve this, an experiment in teaching the topic 'Sequences and Regularities' with open-ended tasks, using a qualitative and interpretative approach, is reported. Data were collected during two class observations, from two interviews and by analysing the activities of the students. An exploratory task was chosen in the first lesson and a investigative one in the second. One month separated the two lessons, and during this time the teacher read and discussed texts on mathematics communication. Observation of the first lesson showed that the communication in the classroom was mostly focused on the teacher, which provided little student-student and student-class interaction. In the second observed lesson, the teacher changed the attention she paid to what each student said and did, encouraging the students to ask each other and encouraged student-class and the student-student communication.

Keywords: Reform of mathematics programmes; teaching mathematics; open-ended tasks; forms of communication; sequences and regularities.

Introduction

The constant evolution of knowledge in the field of mathematical education determines the changes that are made periodically in maths programmes. In Portugal, the reformulation of the basic education maths programs, which began in 2009/10, is now in the implementation stage and covers all the school years this academic year[1]. The basic education maths

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[1] The Portuguese education system encompasses 12 years prior to entry into higher education, as in most countries. The first nine of these years comprise basic education and the last three are secondary education. Basic education consists of three cycles: the first lasts four years with just a single teacher; the second lasts two years, and the third lasts three years. During these nine years the maths curriculum is same for all students. In the three

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programme that is being overhauled has existed since the early nineties (1990 for the 1st cycle and 1991 for the 2nd and 3rd cycles). The publication in 2001 of the National Curriculum for Basic Education brought changes to the previous programme, especially to the learning goals and objectives and to the way that maths topics are addressed. These changes are justified by the need to update the curriculum to the new ways of developing knowledge about the teaching and learning of mathematics and to improve the coordination between the programmes of the three cycles. The programme begins by presenting the general aims and objectives of the teaching of mathematics, which are the main goals shared by three cycles of basic education. Next, it presents the mathematical topics, numbers and operations, geometry, algebra, organisation and data processing.

The present reformulation focuses on the organisation of maths topics that link together the different teaching cycles and the methodological guidelines for teaching these topics; the emphasis was on the cross-disciplinary aspects to be developed over one school cycle – resolution of problems, mathematical reasoning and mathematical communication. Regardless of the study topic, the teaching of mathematics currently recommends the use of strategies which value student activity over a teaching process that is essentially centred on the activity of the teacher, where students mainly listen to and do what the teacher asks (Nicol, 1999, NCTM, 2007). But the conceptions of the teacher about the act of teaching go hand in hand with the curricular reforms (Ponte, 1992), conceptions that are very often focused on teacher authority in validating what happens in the classroom. When appraising what the student says and does in classroom activities, the how the teacher stimulates and manages mathematics communication is paramount. Current methodological guidelines for the 3rd cycle program are that the teacher should present different types of tasks that enable the "comparison of results, the discussion of strategies" (Ministry of Education, 2007, p. 8). Teaching strategies therefore involve engaging the students in activities of analysing, doing, listening, reflecting, arguing, and discussing. Such activities affect how teachers evaluate students' reasoning and encourage them to analyse and respond to other students' reasoning, which relates to how mathematical communication is promoted during classroom work:

> The creation of adequate opportunities for communication is assumed to be an essential part of the work being done in the classroom. (...) Students compare their problem-solving strategies and identify the arguments made by their colleagues through oral discussion in class. They have the opportunity to clarify and explain in more detail their strategies and arguments through written work. (Ministry of Education, 2007, pp. 8-9)

Mathematics communication is essential to enabling students to understand about processes, discussions and decisions that are made. However, the achievement of curricular rules depends on how they are interpreted by teachers and on how they adjust them to their own conceptions on the act of teaching. In order to see to what extent the methodological guidelines are produced in practice, we seek to ascertain how maths teachers promote mathematical communication in the classroom through open-ended tasks.

Communication in mathematics classes

Taking the classroom as a special place for relationships between students and between them and the teacher, the way that this relationship is promoted becomes fundamental in the development of the teaching and learning process. By regulating the social interactions that are generated in the classroom, communication enables the sharing of ideas and clarification of mathematical understanding. Here we have a perspective of teaching that

years of secondary education, where students begin to be routed to a group of higher education courses, the maths curriculum varies according to whether courses in sciences, humanities, arts or technology are followed.

stimulates students to explore and make sense of the mathematical activities that are developed (Brendefur & Frykholm, 2000; Nicol, 1999). The way in which the teacher promotes verbal or written communication determines how the students voice their doubts and justify their ideas. Sharing and comparing the processes results in ways of thinking that promote the significance of mathematical concepts. When the students establish conjectures and discuss the activities with their colleagues, new collaborative knowledge is developed, which ensures that mathematics is seen as a normal human activity (NCTM, 1994).

Brendefur and Frykholm (2000) classify classroom communication as uni-directional, contributive, reflective and instructive. In uni-directional communication "teachers tend to dominate discussions by lecturing, using essentially closed questions. They create few opportunities for students to communicate their strategies, ideas and thinking" (p. 126). In contributive communication the teacher gives the students opportunities "to discuss mathematical tasks with one another, present solution strategies, or help each other to develop solutions and appropriate problem solving strategies" (p. 127). In reflective communication what the students and teacher do "becomes the subject for discussion. Reflective discourse often occurs when students try to explain or refute conjectures offered by their peers" (p. 128). In instructive communication the interactions that occur in the classroom help the students to construct and modify their mathematical knowledge. By verbalising their ideas, the students allow the teacher to understand their thinking processes, their effectiveness and limitations, to alter the way the lesson develops and to draw conclusions for future situations. Apart from uni-directional communication, the other types describe forms of communication to stimulate students to share their ideas, their thoughts, conjectures and mathematical solutions. This is precisely the direction indicated by the new basic education mathematics syllabus when it recommends that "students must be able to express their ideas and to interpret and understand the ideas that are presented and participate constructively in discussions about ideas, processes and maths results" (Ministry of Education, 2007, p. 8).

As for asking questions, the NCTM (1994) considers that the questions that the teacher formulates help students to make sense of their activities, to be able to decide whether something that is or is not mathematically correct, to speculate, argue about and resolve problems and to link mathematical ideas and applications. Moyer and Milewicz (2002) identified various strategies for questioning that the teacher can adopt: (1) *follow the questions as planned*, whereby the teacher passes from one question to another with little consideration for the students' answers; (2) *teach and transmit*, whereby the teacher plants questions to direct the students' answers and stops asking questions in order to teach the concept to be tackled without encouraging the students to think or frame a response; (3) *ask questions and give follow up*, whereby the teacher uses different types of questions to find out more about the ideas of the students and to meet their questions with other relevant questions, thus giving them the idea that their response is still open for discussion; (4) *only question a wrong answer*; (5) *non-specific questioning*, when the teacher follows up the students' answers but with questions that indicate a lack of specificity; and (6) *competent questioning*, when the teacher listens to the students' answers and uses them to gather information about their way of reasoning.

The use of each of these strategies shows the importance that a teacher gives to questioning in the activities carried out in the classroom. Besides the right questions at the right time, Nicol (1999) says that teachers should know how to listen to their students – by paying attention to their words and trying to understand their contributions – and to respond to their actions constructively. A good question represents the difference between constraining students' thinking and encouraging new ideas, and between their retaining trivial facts or constructing meanings (Moyer & Milewicz, 2002).

The nature of the tasks in promoting communication in mathematics

One good way to encourage maths communication is to provide the students with a learning environment that arouses their active participation. One way of doing this is to use challenging tasks (Ponte, 2005). Stein and Smith (1998) draw attention to the importance of choosing tasks that challenge the students to think, justify, explain and find meaning and which stimulate them to make connections. The NCTM (1991) takes the same stand by recommending that the tasks permit students to actively "explore, formulate and test out conjectures, prove generalisations and discuss and apply the results of their investigations" (p. 148).

The nature of the tasks can have implications for how students are involved in the construction of their mathematical knowledge. Ponte (2005) distinguished tasks according to their degree of difficulty (low/high) and their structure (closed/open). Though exercises and problems may be of a closed structure, they will differ in their degree of difficulty. Exercises have a low degree of difficulty, which appeals to the mechanisation and repetition of the processes in pursuit of the intended response. Problems have a higher level of difficulty since they translate non-routine situations for which students do not have an immediate solution process and which can be solved by various methods. These characteristics are also present in investigative task that - according to Ponte - requires students to participate in the "specific formulation of their own questions to be solved" (p. 15), to search for regularities, establish and test conjectures, argue and communicate their processes and their conclusions.

The tasks in which the students carry out a set procedure that is memorised in a routine way are, for Stein and Smith (1998), much less rewarding than the tasks that challenge the students to establish connections between mathematical concepts, to reason and to communicate mathematically. Osana, Lacroix, Tucker and Desrosiers (2006) stress the use of open-ended tasks which favour students' involvement in class activities and encourage them to explore and investigate, increase their motivation for generalisation, look for models and links, communicate, discuss and identify alternatives.

However, the selection of tasks does not in itself guarantee effective teaching. Teachers are crucial to determining the "aspects to be underlined in a given task; like organising and guiding the work of the students; what questions to ask, so as to challenge the different levels of skills of the students" (NCTM, 2007, p. 20). It is important for the students to "work on mathematical tasks that set up relevant subjects for discussion" (NCTM, 2007, p. 66). Discussion is thus the next step after the implementation of the set tasks, thus making it possible for the student to think, rationalise and communicate mathematically. Stein, Engle, Smith and Hughes (2008) set out five practices that promote discussion:

> (1) *anticipating* the students' likely answers to cognitively demanding mathematical tasks; (2) *monitoring* the students' answers to the tasks during the exploratory phase; (3) *selecting* some students to present their mathematical responses during the discussion phase; (4) *intentionally sequencing* the students' responses; and (5) *helping* the class to make mathematical connections between the students' different responses. (p. 321)

These discussion practices contribute to teachers using the students' answers, so as to develop the mathematical understanding of the class. It is within this framework that this study intends to analyse how maths teachers, promote mathematical communication in the classroom through open-ended tasks, during the implementation of the methodological guidelines for the current maths programmes of the third cycle of basic education.

Method

This study is about an experiment devised by two teachers, Mariana and Inês, on the methodological guidelines resulting from the reformulation of the basic education mathematics programme. To increase the implementation of these guidelines, the Ministry of Education created a national network for monitoring it, which consists of university lecturers and teachers representing a group of schools from the same geographical area. These representatives meet regularly with teachers from schools in their area to review and discuss the theoretical assumptions that underlie the changes made in the basic education mathematics programmes. This move had an impact on the professional practice that teachers develop in their schools. The individual work has led to work on an equal footing in the preparation, observation and discussion of lessons, sharing the experiences with their students and discussion of texts on the field of mathematics education. This is context of the work that Inês and Mariana undertook to implement the curriculum in a 7th year class. Mariana was the class teacher and Inês the representative of the schools in their geographical area; both have 13 years teaching experience.

When implementing the methodological guidelines of the revised programme, they paid special attention to the relationship between communication in the mathematics classroom and the tasks the students are set. They therefore chose a topic, *Sequences and Regularities*, for which the new programme specifies a different approach from the previous one. The two teachers together prepared two classes, the first and last on this topic, with open-ended tasks. This topic deals with the general term of a numerical sequence, representation and algebraic expressions.

In the first class, Mariana implemented an exploratory task, while the second class involved a task of an investigative nature. The exploratory task – *V flight* – provides patterns with geometric figures that change at each position according to a rule. The work might be very intuitive at first, describing her natural reasoning, with the use of diagrams, the submission of calculations or the use of symbols. Students can use different strategies to characterise a next term: analysis of the previous figures, analysis of regularity in the associated numerical sequence and decomposition of the figure into parts. When characterising a distant term, students can compare the figure number with the number of points in this figure. The investigative task - *Explorations with numbers* - lets different paths be followed to obtain various regularities and numerical relationships. Students are challenged to hypothesise, test and reformulate their conjecture and generalise. This task promotes written communication as students are asked to describe the regularities identified using natural language and mathematical language. The task provides an opportunity for students to express themselves orally in student-student dialogue on the regularities found, when they work in pairs or groups. Students have to indicate clearly and use a mathematical language appropriate to their findings so that they all understand and can verify that these are always valid. They can also see if the same conclusions are reached, if other ones are reached, or if, based on the findings of their colleagues, they can identify new regularities.

The two classes were observed by Inês, whose attitude was that of non-participant; she focused on the interactions between Mariana and her students during the collective discussion. The time between the two classes was about a month, to ascertain: (1) the development of student participation in classroom discussions and their involvement in learning the sequences and regularities topic, and (2) the teacher's progress in monitoring these discussions, after meetings with Inês when they read and discussed texts on mathematical communication in the classroom.

Following a qualitative interpretative methodology, data were collected through two audiotaped interviews that Inês held with Mariana - one before the experiment (I1) and one after (I2) -, the observation of two classes by Inês (CO1 and CO2), recorded on video, of the discussion of these classes (DCO1 and DCO2) and activities produced by the students. From the analysis of data collected by these methods, the information was organised thus: (1) Mariana's class involving an exploratory task; (2) Mariana's class involving an investigative task; (3) Mariana's views about the influence of the tasks on classroom discussion.

Results

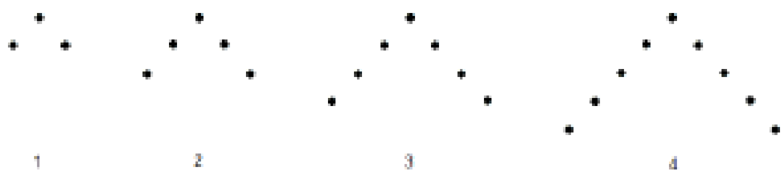
Mariana has been a teacher of mathematics for 13 years - a profession that she thought of following in her ninth school year, as she very much liked this subject. In the current academic year (2009/10) her job was to co-ordinate the third cycle in the implementation of the Mathematics Syllabus of Basic Education (MSBE) in her school and to teach a seventh year class and an education course class. Her 7th year class consisted of 13 boys and 6 girls. It was an uninterested class; 47% of students had failed mathematics at the end of the 1st period.

From her professional career Mariana highlights the moment when the test became mandatory for 9th year students. She explains this because she sees exams as a way to regulate the practice of maths teaching and to encourage varying the type of tasks. For example, she says that prior to mandatory examinations the tasks that prevailed were mainly "exercises" (I1). Realising that national exams have open-ended tasks, she saw that "there could not be more of the same, because the exams involve more than just exercises" (I1). With regard to the changes that have occurred in the pedagogical practices of teachers she stresses that "the collaborative work that has emerged over the past three years and the receptivity of teachers has opened the classroom door to other colleagues" (I1). This year, more than any other, she worked a lot with her colleagues. Thanks to the implementation of MSBE, she met periodically with her colleagues, who are also teaching the seventh year. At these meetings they prepared worksheets and tests, studied and defined strategies and debated the difficulties encountered in implementing them.

One of Mariana's classes involved an exploratory task

To start the topic of *Sequences and Regularities*, Mariana selected the task 'Flight in the V (formation) of ducks' because it allows: (i) checking if a number is a term in a sequence, (ii) determining the order of a known term, (iii) understanding the notion of a general term of a sequence, and (iv) formulating and testing conjectures.

In the sequence that follows each figure represents a flock of ducks and each dot represents one of the ducks in the flock. Here are the first four terms:



Answer the following questions and state your reasoning using words, diagrams, calculations or symbols.

1.1. How many dots does the next figure of this sequence have?

1.2. How many dots does the hundredth figure (term of the order 100) of this sequence have?

1.3. Is there a figure with 86 dots in this sequence? If there is, indicate the order to which it corresponds.

1.4. Is there a figure with 135 dots in this sequence? If there is, determine the order to which it corresponds.

1.5. Write a rule for determining the number of dots in each figure of this sequence.

1.6. Write an algebraic expression that expresses the rule described in the previous question.

Figure 1: Exploratory task given to the students, about sequences and regularities.

The teacher started the class by organising the students into groups, and by instituting the following rules: "tell your ideas to each other (…) at the end we are going to discuss the conclusions and share the different strategies" (CO1). Next, she gave the task to all the students and delivered to each group an OHP transparency on which to record their responses. The students began to solve the tasks without any explanation from the teacher, who was busy with the management of the group work: "exchange ideas, explain your reasoning and only then write your answers on the transparency" (CO1). When the students showed they were having difficulties, Mariana asked questions to guide them in the activity that they were carrying out, as is shown in the following example:

Student: The rule is to add 2 to the previous figure.

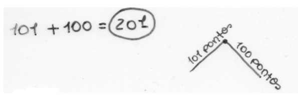
Teacher: In fact, adding 2 to the number of dots in the previous figure does allow you to discover the number of dots in the next figure but will this be a practical strategy to find the number of dots in the hundredth figure? Look at the various figures. What other characteristics do they have? What can we use to represent the given information? (CO1)

In the discussion phase, when the spokesman of one group presented its solution, the rest of the students in the class did not intervene spontaneously. The attention of the teacher centred on the answers that were given by each group spokesman and she confirmed them with statements like "very good" (CO1). When the answers were wrong, Mariana asked another group for its answer. After the presentation of the solutions by the group spokesmen, the teacher would interpret the solution by repeating what the student had explained:

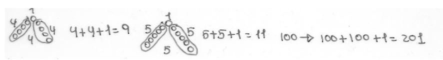
Student: I made 101+100 = 201. The explanation is that in Figure 1, 2+1 = 3; in Figure 2, 2+3 = 5; in Figure 3, 4+3 = 7; thus in Figure 100, 101+100 = 201

Teacher: Do you see what he found out? Do you see how? Perhaps it was geometrical, wasn't it? Is there another group that also saw this characteristic? Who did? Was it you Tiago? Did anyone see it another way?

Diana: We drew a diagram. On one side we put 101 and on the other 100



Teacher: This group used the same reasoning only they drew a figure. In Figure 100 they imagined that on one side of the V they had 101 and on the other they had 100. So $100+100+1=201$. (CO1)



The determination of the number of dots in Figure 100 helped some students to formulate and test conjectures. Only two groups indicated the general term ($n \times 2 + 1$ and $c + c + 1$). In this generalisation, one of these two groups turned to symbolic representation and the other group expressed their reasoning through symbolic representation and a diagram.

Mariana's class involving an investigative task. For this class, Mariana selected the task *Explorations with numbers*[2] so that she could encourage mathematical communication between the students in the discovery of numerical regularities and relationships.

Look at the following table:

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
…	…	…	…

Answer the following questions. Give your reasoning using words, diagrams, calculations or symbols.

1.1. Continue the representation of the table presented above until you obtain the number 40.
1.2. Assume that this table continues indefinitely. Identify the regularities that you manage to find.
1.3. In this table can you predict in which column you will find the number 64? And in which line?
1.4. Can you predict in which column you will find the number 99? And in which line? Explain how you proceeded.
1.5. Taking any number, can you predict in which column and in which line it will be found in this table? Explain your answer.

Figure 2: Investigative task proposed to the students about sequences and regularities.

With this task Mariana wanted the students to guess in which line and in which column a specific number would be found, so that they would manage to generalise for any number. As she wanted to involve the students in the discussion about the task, Mariana set the rule that whoever "does the presentation must involve the others and these others must ask questions, request explanations and, if they do not agree with what is being said or wish to add something, that they should intervene" (CO2).

The first regularity encountered by the students was the multiples of 4, which encouraged the students to "look for other multiples" (CO2). When she found that they were only concerned with discovering multiples, the teacher encouraged them to look for "another type of number, one that we have already talked about in class (…) you have to communicate, describe the regularity encountered and write the algebraic expression" (CO2). When presenting their activities, the students identified some regularities and showed the general term of the sequences that they had found:

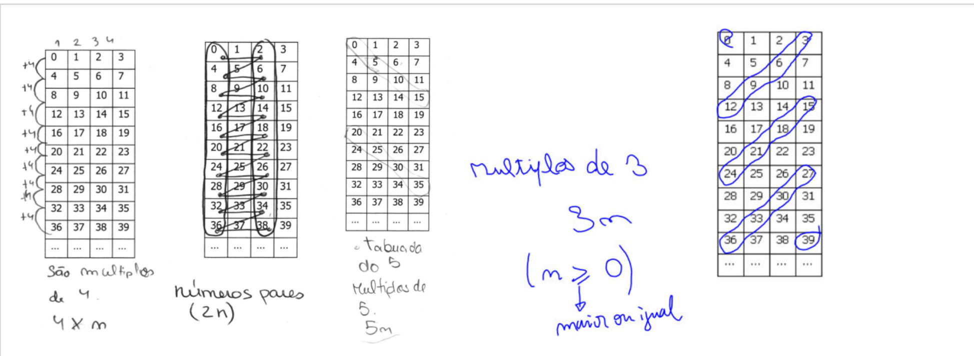
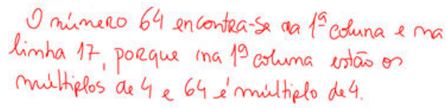


Figure 3: Students' solution of some questions from the *Explorations with numbers* task.

[2] Ponte, J. P., Branco, N., and Matos, A. (2009). *Álgebra no ensino básico*. Lisboa. Ministério da Educação, DGIDC.

In relation to the question "Can you predict in which column you will find the number 64?" one student gave his group's answer on the interactive board, which exemplified the sort of communication that livened up some parts of the class:

Diana: The number 64 is in the first column and on row 17, because the multiples of 4 are in the first column and 64 is multiple of 4



Teacher: Read Diana's answer.

Student: Why is it on line 17?

Diana: I always added 4 to the numbers in the first column and arrived at 64. It gave 17.

Student: It would be 4x16.

João: I did the table. I was writing the numbers and I got to 64.

Teacher: This is not the purpose of the question; it is to predict and not to confirm. Did anybody find a strategy for prediction? Somebody did? Did anybody manage to predict why it is line 17? I will have to give a hint….

Student: Wait a minute…

Students: I know! I know!

Teacher: Work on your idea. Think better! Keep thinking and check your strategy!
[Mariana gave them a little more time to think]

Teacher: Diana, have you got it yet?

Diana: No.

[The teacher asks Diana to sit down and lets Rui speak]

Teacher: Those who do not understand ask Rui.

Rui: 4x16 gives 64. But the 4 isn't on the first line, 4 only comes on the second line.

Students:I don't understand.

Teacher: You didn't write what you've said …

Rui: It's the way it is, 4x16 = 64. Since 4 doesn't come on the first line, this gives 17.

Teacher: Who doesn't understand? Ask Rui questions. Those who already know can help Rui to explain (the answer).

Renato: I don't understand your explanation.

Rui: 64 is in the first column. But, as the 4 isn't on the first line, you have to add 4 by 4, 16 times from 4 [the student exemplifies with gestures next to the table]. Afterwards you have to add the first line. Thus it's on line 17.

Teacher: Anybody want to add anything? Is it clear now?

Students: Yes.

Teacher: Good… but we need to complete the answer. You get there without me giving a hint. (CO2)

The teacher tried to get the students to ask the colleague that presented their answer one of the specified questions. The question was a form of contributive communication sustained by student-student, teacher-student and student-class interaction. There were only a few spontaneous interventions from the students, which raised the question: what happened to the requested explanation? The students begged their colleague for explanations instead of asking Mariana.

Synthesis of Mariana's two classes

The way the communication was promoted in the two maths classes differs, as noted in the interaction that develops between the teacher and students in each class.

Table 1. *Promoter of the communication in the classroom.*

	Lesson 1	Lesson 2
Initiations		
Request response	Teacher	Teacher and students
Request for explanation	Teacher	Students
Answers		
Answer	Students	Students
Explanations	Students and teacher	Students
Reconceptualisation		
Reaffirm	Teacher	Students
Expand	-	Students
Reformulate	-	Students
Validate	Teacher	Students and teacher

In the first class, when students presented their activities, their colleagues generally did not participate willingly. The teacher tried to get the students to justify their answers. But, was Mariana who validated almost all the answers and who interpreted the students' presentation to the class. After a student gave his explanation the teacher tended to reaffirm what this student said. When questions arose, students directed them at the teacher.

In the second class, the students were more attentive to the presentations of their colleagues and student-student interaction was more frequent. The divergence of answers that the task provided meant that students gave presentations that had not been explained. The teacher took care not to validate the answers and so created space for the students to do it. When the students asked the teacher, she sent the question back to the class, which meant that they sometimes addressed and asked colleagues who were presenting their activity.

Mariana's views about the influence of the tasks on classroom discussion. In terms of the methodological guidelines that have emerged from the reformulation of the Mathematics syllabus for Basic Education, Mariana pointed out "the type of tasks that are different from those usually implemented and the topics that are approached in an exploratory way by discovery" (I1). For Mariana, the effect of the exploratory tasks on the learning of the students raised "many doubts about whether we would get better results, whether the students would be more competent mathematically (…) we have a lot of work and little supervision" (I1). She assumed that she would not always make "the students interact with each other, perhaps because they aren't used to it" (I1). Before the experiment, Mariana recognised that the form of communication that predominated in her classes was uni-directional communication sometimes with interpolations when the students would be asked "to justify their reasoning and explain how they think" (I1).

When analysing the first class observed, Mariana identified critical aspects of her action, such as a tendency to repeat what the students said and did and the difficulty of encouraging the students to discuss their activities with one another:

> The students explain and there's something I always do, but I don't know if it's good or bad; the students explain and I repeat their explanation, I don't know if I should do that. Another thing that I think is that I don't promote student-student communication, which I think is very difficult, but some people think it can be done. Student-class and student-teacher communication exists, but student-student - the type where one student puts their hand up and asks another – there's none of that. It may have happened in groups but in groups it is very difficult to evaluate, we would have to monitor each group closely. They put their hand in the air to give their answers or when a student answers badly, but they don't ask questions. I have to improve student-student communication. (DCO1)

The teacher recognised that the students did not question their colleagues' answers and she questioned the way that she promoted the confirmation of the students' answers: "I always confirm, never ask if they agree... I should ask for another strategy leaving out the previous automatic ratification" (DCO1). She was aware, above all, that she asked questions from her point of view and that she gave little time for the students to respond to what she ended up doing. In the class discussion of a presentation by the spokesman of one group, Mariana was able to solicit an explanation from this group and later on she was able to direct another group to present an explanation that would contradict or supplement the one given.

From the analysis of the second class, Mariana identified the initiative of the students in stimulating communication between them without her having to intervene, which in her view she did not manage in her previous classes: "I had the students communicating more student-student" (DCO2). Although she had used easy questions, she recognised that the question of generalisation was only understood by some students, which she would have widened if "in the previous questions, she had prepared them better for managing to generalise" (DCO2). Time limited her action and this prevented her from "exploring what the students did a little more" (DCO2). When comparing the two classes, the teacher considered that "in the first the discussion was very much centred on me, it was me that confirmed" (I2), while "in the second class I gave more opportunity to the students but it made the class more time-consuming" (I2).

Including open-ended tasks in the classroom has implications for the care and time necessary for their preparation. Carrying out of this type of task ensured that Mariana paid heed to what the students said and did and she had to look for ways of involving them in class discussions:

> I liked to prepare the class, where I would have a place for discussion and not simply the preparation and solving of the task, more frequent in my day-to-day work. I was aware of the importance of frequent class discussions between the students and paid attention to my efforts to promote these discussions. During classes I asked myself: Are they communicating among themselves? What questions should I put? And if this happens, what should I do? Should I wait a bit longer? What example should I choose to stimulate discussion? (I2)

Besides the structure of the task, the rules that the teacher established for carrying out the student activities and the conceptions she had about the teaching of mathematics tended to influence how the students engaged in the class activities:

> All the same it was very difficult to involve many students in mathematical discussions and, while there was some progress between the first and second classes, the students did not communicate with each another but limited themselves to setting out their ideas. It was me that was always intervening, essentially by asking pointed questions and finishing off by confirming the answers. The student interventions coming from the class were short (presentation of their answers) and limited, since the interaction was predominantly from teacher to student. However, there were times when the students

> questioned their colleagues and gave valid reasons, leaving me in a less prominent position. All the same, the students' contributions came close to influencing the course of the lesson with some inspired discoveries and questioning about them by the others. Inquiry questions predominated in discussions about the task of this class up to question 1.5, which is not my normal practice, while for question 1.5 focusing questions were asked, which is more normal for me. (I2)

The completion of the two tasks in the classroom enabled Mariana to perceive the influence that her conceptions had on the way that she promoted communication: uni-directional communication - with little space for the students to intervene and, when they did, it was to present answers - gave way to contributive communication during the presentation and discussion of the students' solutions, which tended to influence the course of the class. Her openness to innovate in her practice contributed to this change and it also helped her to read and discuss with colleagues texts about the didactical aspects of teaching.

Discussion

Of importance to the translation of the methodological guidelines of the current school syllabuses are the nature of the tasks that teachers should adapt for their classes, and particularly the attention to be given to student activities, as this gives an understanding of the way others think. The conceptions that teachers develops in their professional career about the teaching of mathematics tend to hamper the implementation of these guidelines (Ponte, 1992). Willingness to innovate in teaching practice and a critical analysis of it help to overcome some obstacles, as observed in the teaching practices of Mariana in relation to how she fostered communication with her students. Although she considered that discussion of classroom activities is one factor that stimulates student learning, the teacher did recognise the difficulty students have with the presentation of alternatives to the proposals presented by their colleagues. This difficulty tends to be due to the habits that students develop in learning environments where the authority of the teacher in the management of classroom activities prevails (Moyer & Milewicz, 2002; Nicol, 1999). It is the belief that teaching is a uni-directional process of transmitting information to students in a way that enables them to reproduce what the teacher says and does (Brendefur & Frykholm, 2000).

Before completing the exploratory task, Mariana questioned the importance ascribed by the methodological guidelines to this type of task in student learning, because of the time it requires, which would indicate a preference for repetitive tasks of a lower cognitive level (Stein & Smith, 1998). In this way she became aware of her fears about organising the students in groups in her classes. Such fears were overcome in the class in which she proposed the exploratory task covering the topic '*Sequences and regularities*.' As the students were not used to working in groups, the teacher stated some rules about how the students should communicate their ideas to the others. She herself realised that it was not the rules that she defined or the nature of the tasks that were chosen that really altered the atmosphere in the classroom. In the first lesson observed, teacher activity prevailed to the detriment of student activity. When a student from one of the groups presented its solution, the others did not intervene. Mariana tended to explain what the students were doing by repeating what they said, and to ask planned questions (Moyer & Milewicz, 2002). After the class with the exploratory task, the teacher recognised in this class that she repeated what the students said and did. She did this for the benefit of students who did not question their colleagues who had presented their solutions. Although Osana et al. (2006) consider that tasks of an open nature stimulate students to engage in class activities, Nicol (1999) stresses the importance of teachers knowing how to listen to their students in order to encourage them to discuss the classroom activities. Only then, as suggested by Moyer and Milewicz

(2002), can the teacher use student responses to collect information about their way of thinking.

The reading and discussion of texts about mathematics education with other maths teachers made it clear that Mariana needed to hear more about what her students said and that she needed to develop in them the habit of asking their colleagues when they had doubts or had other strategies for solving the task. This is what happened in the class with the investigative task. The students put questions to their colleagues, who answered, and when they had doubts they did not put them to the teacher but to their colleague who was presenting his group's solution, with the aim of understanding the answers he gave. As the prevailing forms of communication tend to move away from uni-directional, students contribute to the course of the class and give meaning to learning (Brendefur & Frykholm, 2000). These are the guidelines that are emerging from the current programme of the 3º cycle (Ministry of Education, 2007).

Comparing the two classes, the teacher confessed that in the first class the students did not communicate with one another; there was no direct communication between them, and that she herself neither give them time nor stimulated student-student communication. She realised the need to try to get the students to present their ideas and to ask each other questions by taking on the role more of a moderator (Stein et al., 2008). Mariana admitted that it is "very difficult to involve many students in mathematical discussions but I noted some improvement from the first class to the second one, when there were times when the students questioned colleagues and confirmed reasoning, while I stayed more in the background" (I2). In the second class the teacher felt that "the contributions of the students came closer to influencing the course of the class with inspired discoveries and questioning from the others about these discoveries" (I2). The change in the way that she encouraged student communication gave the impression that it was due, as advocated by Stein and Smith (1998), to the higher cognitive level of the task that she proposed, which stimulated the discussion and formulation of conjectures. But also it was due to the attention that the teacher gave to the students' answers. Consideration of what the students say and do must become part of a classroom culture that is nurtured in the earliest of school years and should persist in the more advanced years. Only then will the students understand that their involvement in class activities is not only enriching their own learning but it is also enriching the learning of their colleagues. The discussion of texts on mathematical communication with a colleague and the divergent nature of the open-ended tasks played a major part in bringing about this change.

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