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Using a Touch Point Instructional Package to Teach Subtraction Skills to German Elementary Students At-Risk for LD

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Abstract

This study replicates and extends prior research on multisensory mathematics instruction (Grünke et al., 2018) by integrating a touch points strategy, performance feedback, reward system, and a reinforcing card game into an instructional package. A multiple baseline design across participants was used to evaluate the effects of the touch points package on the subtraction skills of four German female first year students at-risk for learning disabilities. During intervention, the students were administered eight to eleven treatment sessions to learn how to subtract a one-digit subtrahend from a two-digit minuend up to 18 crossing over the tens barrier. Results indicated that all students made substantial increases in their subtraction performance during intervention. Student performance improved from 0 to 2 out of 10 math problems solved correctly during baseline to between 8 and 10 problems correct by the end of the intervention. Effect sizes observed across the four participants indicated the effectiveness of the intervention ranged from high to very high.

Keywords:

TouchMath, Subtraction Skills, Single-Case, Elementary, At-Risk, Learning Disabilities

Introduction

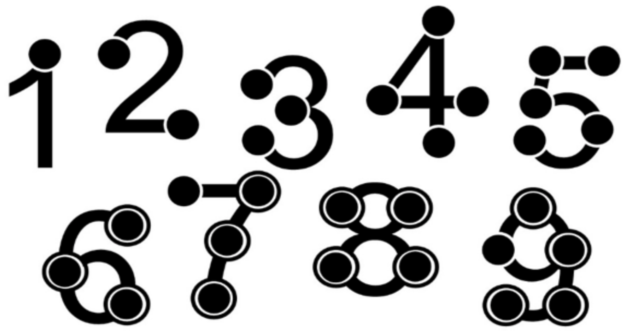
Mathematical difficulties are a pervasive problem for children and adolescents (Lein et al., 2020). About 17% of elementary and secondary students exhibit some form of mathematical difficulties and perform poorly, frequently well below school-grade expectations (Geary, 2015; Mazzocco & Vukovic, 2018). Many of these students, approximately 4%-7% of the school-age student population, will be later on identified with a learning disability (LD) in mathematics (Butterworth, 2019; Geary, 2011). Struggling with basic numeracy during the first school years are signs of mathematical difficulties (Stock et al., 2010; Tolaret al., 2016). Deficits in early math skills typically compound into further mathematical difficulties in the upper grade levels, and these difficulties often extend into adulthood (Bryant et al., 2020, 2021; Nelson & Powell, 2018; Powell & Driver, 2015; Powell et al., 2020). Thus, not addressing mathematical learning problems in the school context have negative long-term effects not only on the students' academic success, but also on later opportunities for vocational training, employment, and overall quality of life (Fischer et al., 2013; Geary, 2013; Kaufmann et al., 2020; Ritchie & Bates, 2013).

These concerns make particularly clear the imperative need to identify effective instructional practices (Mazzocco et al., 2018) that enable teachers to empower students' mathematical basic skills while helping them to overcome their difficulties at an early age (Dennis et al., 2016; Jitendra et al., 2018; Stevens et al., 2018). One such essential skill is the knowledge of number combinations (Kilpatrick et al., 2001). These are simple arithmetic problems (e.g., 4 + 5 = 9; 8 – 5 = 3) that can be solved by counting, applying decomposition strategies, or by automatic retrieval from long-term memory. The most fundamental way to meet this challenge is to use efficient counting strategies. While typically developing children do not need to be taught efficient counting techniques, students with mathematical difficulties do not discover them on their own (Ashcraft & Stazyk, 1981; Geary et al., 1987; Goldman et al., 1988; Groen & Parkman, 1972). Initially, learners use the strategy "counting all" to determine a sum. The application of more sophisticated counting strategies includes the possibility of "counting on" as well as their understanding of the commutative property of addition, which allows counting from the larger addend, regardless of the order they are in the arithmetic task (Baroody, 1995). The expansion of efficient counting and, later on, the acquisition of decomposition strategies, leads to more reliable retrieval of facts from working memory and a higher probability that these are also stored in long-term memory (Ashcraft & Stazyk, 1981; Geary et al., 1987; Goldman et al., 1988; Groen & Parkman, 1972).

Fuchs et al. (2010) demonstrated the effectiveness of a single intervention based on a direct instruction of the principles of strategic counting for students with arithmetic difficulties. In addition, they showed that when this intervention was combined with opportunities of deliberate practice, the positive effects on number combination became even more apparent. Furthermore, the authors recognized that effective instructional interventions as theirs and others in previous studies adhere to the following principles: (1) explicit instruction, (2) minimal learning challenge for students, (3) opportunity for practice, (4) and the use of motivators to help students with motivational and behavioral regulation (Fuchs et al., 2010, p. 98).

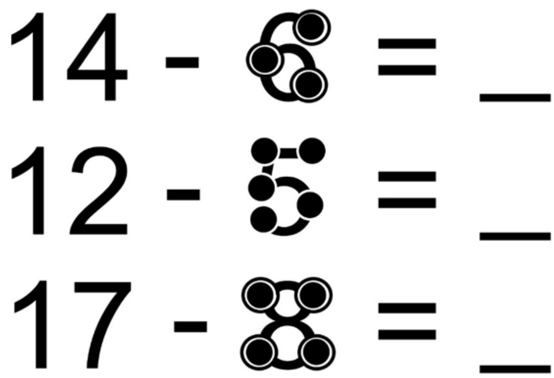
Another research-based intervention that aligns with the principles of Fuchs et al. (2010) is the use of touch points. This strategy fosters effective counting strategies and reinforces understanding of the cardinal concept of numbers in which the visual and haptic presentation of the quantity is used to associate it with the respective number word. The dot-notation approach, in which touch point dots are placed on the digits, was developed by Kramer and Krug (1973). Based on this method, Bullock, Pierce, and McClellan (1989) created the so-called TouchMath concept (see www.touchmath.com), an instructional mathematics curriculum to teach from basic addition and subtraction to more advanced arithmetic skills. Its central concept is that each number, according to its quantity, is illustrated with touch points. Whereas on numbers 1 to 5 shows only single points, on numbers 6 to 9, single and double points are used, accordingly (see Figure 1).

Figure 1
Numerals with Touch Points from 1 to 9



In the touch point strategy, students first learn to touch every point on a digit in a predetermined sequence while counting them aloud, which fosters cardinal number understanding. When solving a single-digit addition task, students use the counting-all strategy by tapping the dots on the summands. Thereafter, for addition tasks, learners are taught first to choose the bigger addend and, count forward while tapping the dots on the second addend. To solve single-digit subtrahend subtraction tasks, students are instructed to count backwards from the minuend tapping on the subtrahend touchpoints to reach the solution (see Figure 2 for some examples from a touch point subtraction worksheet).

Figure 2
Examples of Touch Point Subtraction Problems



Over time, touch points are faded from numbers to promote mental representations of quantities and acquisition of fact knowledge. The procedure of the touch point strategy is not limited to single-digit addition and subtraction; it can also be used for multiplication, division, and double-digit problems. One of the unique advantages of this method is its multisensory nature, which can be especially helpful for students with learning difficulties. Touching the points stimulates haptic perception, counting aloud activates auditory perception, and looking at the dots cues visual sensation (Scott, 1993).

Research on the touch points strategy has largely been conducted in the United States, some studies took place in Canada and Turkey, and one study in Germany (Grünke et al., 2018). Evaluation of the TouchMath strategy for students at-risk or with disabilities in mathematics instruction has focused mainly on addition, generally on single-digit addition (Ellingsen & Clinton, 2017). The investigations on the use of the TouchMath program were predominantly conducted at the elementary level, mostly with students with an intellectual or developmental disability, and very few targeted students with or at-risk for LD. However, there is a dearth of research on the effects of the touch points strategy to assists students with or at-risk of disabilities to acquire subtraction skills.

To date, only two studies were found that examinedthe efficacy of TouchMath for improving students with disabilities subtraction skills (Scott, 1993; Waters & Boon, 2011). Using a multiple probe design across four math skills, Scott (1993) assessed the effectiveness of TouchMath to teach three fourth grade students with disabilities, two with intellectual disabilities and one with LD, two-digit addition with regrouping, subtraction up to 18 with a single-digit minuend, and two-digit number with regrouping. The three students' performance on practice and novel problems was high after the introduction of the intervention for each of the math skills taught. In particular for subtraction skills, the students' performance score increased less than 14% on average during pre-intervention probes for practice and novel problems to over 86% on average in post-intervention probes. In the Waters and Boon (2011) study, three high school students with mild intellectual disability, two of which were also diagnosed with autism, were taught to perform three-digit money subtraction with regrouping using the TouchMath program. The strategy was effective to improve the subtraction skills of the students, with average improvement on performance of 69% to 83% from baseline to intervention. Upon completion of the intervention, one student maintained the subtraction skills over approximately 5 weeks, while another student experienced a substantial gradual decline in performance during 20 days. These studies show promise that the touch point system can assist students with disabilities to learn basic subtraction skills. Given the limited research on the effects of the touch point strategy on subtraction skills for students with or at-risk of LD, more studies are needed to explore its effectiveness for this population.

The purpose of the present study is to replicate and extend a previous experiment by Grünke et al. (2018) to examine the use of a touch points intervention package to teach subtraction skills to four first graders at-risk for LD. This study was aimed to answer the following research questions:

1. What is the effectiveness of a touch point instructional package to solve subtraction problems within 18 for students at-risk for LD?
2. What are the students' attitudes towards the touch point intervention?

Method

Setting and Participants

The study took place in a resource classroom in an urban public school in North Rhine-Westphalia, Germany during the last weeks of the school year. Four female first grade students at-risk for LD with ages between 6 and 7 years old enrolled in the same class at the school served as participants in the study. According to the school's curriculum, students are expected to have mastered the concept of subtraction up to 20 (e.g., 15-8) by the end of first grade, which in Germany constitutes the first year of formal schooling. Prior to the start of the study, the students had received instruction on addition and subtraction up to 20 using traditional instructional methods, however, math class instruction during the duration of the study did not focus on either addition or subtraction skills.

The eligibility criteria to participate in this study required participant students to: (a) be able to count forward to 20 and backwards from 10, (b) be able to count with one-to-one correspondence up to 20, (c) be able to add fluently within 20, (d) perform below grade-level on subtraction as required by the school's curriculum for first graders, (e) consent to take part in the study, and (f) have a high level of school attendance over the last six months. Before beginning the study, the special education teacher administered an informal test designed according to the diagnosis and training sheets by Klauer (1994) to evaluate the addition and subtraction skills of the students in her classroom. Based on the students' assessment scores, a detailed analysis of their addition and subtraction performance in their mathematics workbooks, and attendance records, the teacher and the second author identified four students that met the inclusion criteria.

The first participant, Aylin, was 7.6 years old. Her parents were both from Turkey and Turkish was their primarily language spoken at home. Informal assessments indicated that Aylin was unable to solve basic subtraction problems. She also showed neither a cardinal nor ordinal understanding of numbers, and was also unable to represent quantities or order them in relation to each other. The teacher characterized her as insecure in her mathematical abilities, but she was eager to improve her math skills. Aylin often would get upset when she experienced any kind of failure and frequently cried if she did not succeed on solving a mathematics problem.

Blanka was 6.8 years old and born in the Congo to French-speaking parents. She started learning German when she entered preschool. Even though she had mastered her addition facts through 20, she was unable to solve subtraction problems. However, she had a fairly well-developed ordinal understanding of numbers and was able to verbalize the steps she used in solving different mathematical problems. Blanka did not ask for help whenever she experienced difficulties; instead, she just waited for teacher assistance.

Carla was born in Germany and was 7.1 years old. She was mainly raised by her Turkish grandmother. Carla started to learn German when she was three years old, but still had trouble understanding the language. She had received special language training at her school since she enrolled. Carla had satisfactory addition skills; however, her subtraction skills were lacking.

Dana was a 6.1 years old girl, and her first language was German. She was able to represent quantities up to 20 and describe the steps she took to arrive at answers to various problems in mathematics. Her addition skills were acceptable for a first year student; however, like Aylin, Blanka, and Carla, Dana exhibited poor subtraction skills. According to her teacher she appeared very motivated to work on her subtraction skills, but often would become impatient with herself.

Since all four of the participants were still in their first year of elementary school, they had not yet been officially diagnosed with a disability. However, all the available academic data on their learning aptitude suggested they will soon be identified with some type of learning disability. Furthermore, the German proficiency level of the three second-language participants was neither formally evaluated at the time. According to the classroom teacher, except for Carla, the German skills of the other two students did differ, although not substantially, from those classmates without an immigrant background.

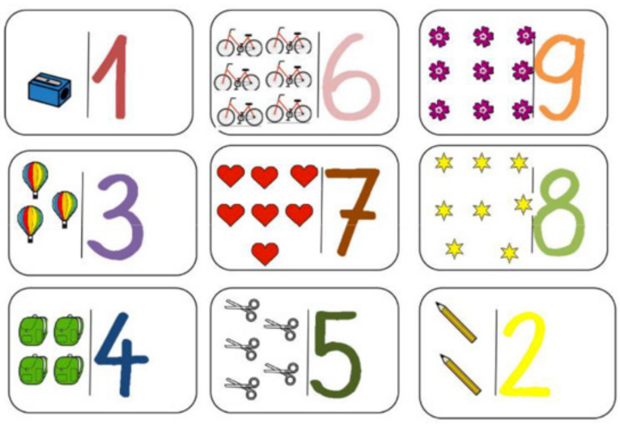
Two female special education graduate students served as interventionists in this study. Both interventionists were in their final months at the university before entering into the probationary teaching period to finish their training to become fully licensed special education teachers. Due to several months-long internships in schools and their jobs as teacher assistants, they both had ample experience working with struggling learners. To avoid conflicts with their teaching schedule at the school, the interventionists took turns to administer baseline and intervention sessions throughout the study.

Materials

Assessment materials included fourteen 10-item subtraction problems worksheets. Each subtraction problem consisted of a two-digit minuend up to 18 and one-digit subtrahend, where the tens had to be crossed to arrive at the correct difference (e.g., 12–8). The pool of subtraction problems meeting the aforementioned criteria were classified by two experienced first grade teachers in three levels of difficulty. All of the fourteen subtraction problems worksheets were designed to have a similar level of difficulty. A stopwatch was used to measure the time during assessment probes.

Intervention materials consisted of 4-inch numerals, dots, and minus signs made out of colorful sponge rubber, a set of cards, stickers, 10-item subtraction problems worksheets with dots in the subtrahend and worksheets without dots. All of the subtraction problems in the worksheets consisted of a two-digit minuend up to 18 and a single-digit subtrahend that required crossing the tens barrier. All worksheets had the same level of difficulty. A set of 1-inch by 2-inch laminated index cards displaying each a digit from 1 to 9 along with as many objects as the cardinality of the number. Finally, stickers with different motives based on characters from various popular cartoon series were used as rewards for performance during intervention.

Figure 3
Cards for the Card Game



Dependent Variable and Measurement

The number of subtraction problems solved correctly on a worksheet within 1-min was the dependent variable. Assessment worksheets were randomly administered without replacement to each student across baseline and intervention sessions. Two special education graduate students blind to the purpose of the study independently scored the mathematics worksheets. Inter-rater reliability was conducted on all of the assessment probes for each participant, and was 100%.

Experimental Design and Procedures

A multiple baseline across subjects design (Gast et al., 2018) was used to examine the effectiveness of the touch point instructional package to improve the subtraction skills of four elementary students at-risk for LD. The study was conducted over three weeks spanning across 14 sessions altogether for each participant. The intervention starting points were randomly assigned to the students to enhance the internal validity of the experiment (Tate et al., 2016), and staggered with baseline probes varying between three and six. Thus, the number of intervention sessions ranged from eight to eleven.

General Procedures. In each session, one of the interventionists took a student to the resource classroom and worked individually with the student for 20 minutes. After completion of the session, the student was given a 10-item subtraction problems assessment worksheet and was encouraged to work on the problems as fast and accurate as possible. Then, the interventionist started the stopwatch. After 1 min, the student was asked to stop working in the problems and praised for their effort.

Baseline. During baseline sessions, the student did not receive any instruction. Instead, the student and one of the interventionists worked together to make handicrafts. To prevent that differences in performance between the baseline and intervention condition might be due to an allocation effect, baseline sessions were set to last 20 minutes. After 20 minutes, a 1-min probe was administered to the student.

Intervention. The interventionists implemented a touch point instructional package that included: the use of the touch points strategy, performance feedback, performance-based rewards, and a card game. The intervention was comprised of six instructional lessons, each lesson taught within a session, followed by one or more independent practice sessions. At the beginning of each intervention session, the student was shown a chart displaying the number of subtraction problems she had correctly solved so far, and were told she would earn a sticker that could be placed on the chart if their performance was at least as good as in the previous session. Intervention sessions ended with a 5-min card game designed to reinforce learning of the touch point notation and counting as well. The card game was played as follows: First, a card was selected at random by the interventionist and given to the student, then the student stated the number of objects he saw on the card (e.g., two pencils, seven hearts) and lastly while touching the objects on the card he counted from the number up and then backwards. Following the end of the session, the student completed a 1-min assessment probe, after which, they received performance feedback, and were rewarded with a sticker if they maintained or improved their prior performance.

In the first lesson, the interventionist taught the student the touch points system using the sponge rubber digits and the dots, one digit at a time. The interventionist presented a rubber single-digit number displaying the appropriate touch points and then modeled how to count the touch points on a single-digit number. Next, the interventionist asked the student to practice placing the touch points on the rubber digit. Afterwards, the student named the number and then tapping on the touch points counted aloud forward from the digit up, following that, the student named the highest number reached and counted backwards while touching the touch points. For example, after placing the touch points on the digit 8, the student named the number 8 and then counted forward using the touch point from 9 to 16; next, he named the number 16 and immediately counted backwards down to 8 while tapping on the touchpoints. The student needed to perform each of these steps correctly on the digit before moving to the next digit. If the student made a mistake, the interventionist corrected the error and prompted her to continue. Three to four rounds of this procedure were required across all the students to learn the touch points on the rubber digits and count forward and backward correctly.

In the second lesson, several subtraction problems presented with rubber digits and dots were used to teach the student the touch point strategy to solve subtraction problems. The subtraction problems consisted of a two-digit minuend without touch points, and a one-digit subtrahend with touch points. To start the lesson, the interventionist showed a subtraction problem (e.g., 13-5) and proceeded to model the steps to solve the problem as follows: First, she started by naming the minuend (13) and then counted down accordingly to the number of dots on the subtrahend to reach the correct solution (12, 11, 10, 9, 8). Then, the interventionist demonstrated the procedure one more time with a second subtraction problem. Next, the student was instructed to solve a different subtraction problem while verbalizing aloud the steps to reach the solution. If the student made a mistake, the interventionist corrected the error and encouraged the student to continue solving the problem. The student practiced the subtraction procedure on at least four more additional subtraction problems, as many as time permitted.

Lessons three and four mirrored lesson two. In the fifth lesson, the student worked on a subtraction problems worksheet that displayed touch points on the subtrahends (see Figure 2). The student was asked to state aloud the steps she applied to solve each of the problems. If she had difficulties solving a problem, assistance was given by the interventionist, as needed. In the sixth lesson, the student worked on a subtraction problems worksheet without touch points. The student was instructed to draw the dots on the subtrahend before proceeding to solve a problem, and verbalize the steps to reach the solution. The interventionist constantly monitored the student's work on the practice worksheet. Help was provided when the student made an error when either drawing the touch points on the subtrahends or applying the steps to solve a problem.

During the independent practice sessions, the student was required to independently solve the subtraction problems on the worksheet that did not display touch points. The student was instructed to work through the problems to find the solution without drawing the touch points on the subtrahends. They received assistance if they asked for help from the interventionist.

Interventionist Training and Procedural Reliability

The interventionists received three 45-min training sessions conducted by the second author before the study began. Training on the procedures to teach the interventionists the touch point instructional package included explicit instruction, modeling, guided practice, and corrective feedback. In addition, the interventionists received training on the administration of the assessment probes. Baseline and intervention sessions followed a detailed step-by-step script to warrant a consistent implementation of the procedures. During each session, the interventionists marked on a checklist the steps they completed as they delivered the procedures. Both interventionists reported they completed each and all of the steps on the procedural checklists. Across all phases, the second author and the interventionists stayed constantly in contact by e-mail and phone to ensure the procedures were delivered as intended.

Social Validity

A teacher's assistant individually interviewed the four students after the intervention ended to capture their views and perceptions on the touch point instructional package. The student interview consisted of the following questions: (1) Did you enjoy calculating with the touch points? (2) Was calculating with the touch points easier for you than without them? (3) Would you prefer to continue calculating with touch points? And (4) How did you like getting constant feedback about your performance? Student answers were recorded, transcribed verbatim, and then analyzed in accordance with a simple approach outlined by Tesch (1990).

Data Analysis

The data analysis of the study includes visual analysis and descriptive statistics for each of the students across phases (Lane & Gast, 2014). Level, trend and stability was estimated for each condition. The stability criterion was set to 80% of data points falling within +/- 20% of the median (Lane & Gast, 2014). Furthermore, two commonly used non-overlapping effect sizes were calculated for each participant: percentage of non-overlapping data (PND) and Tau-U, to measure the effects of the intervention.

PND summarizes the percentage of intervention scores that exceeds the most extreme baseline score in the therapeutic direction (Scruggs et al., 1987). Participants' PNDs were averaged to obtain an overall PND. A PND over 90% suggests the intervention was very effective, from 70% to less than 90% effective, from 50% to less than 70% questionable, and below 50% ineffective (Scruggs & Mastropieri, 1998). Tau-U is a non-parametric effect size that can be interpreted as the percentage of improvement from baseline to intervention (Parker & Vannest, 2009; Parker et al., 2011). Tau-U computes a measure of the non-overlap between baseline and intervention phases while taking into account the intervention phase trend and also can control for monotonic baseline trend (Parker et al., 2011). Tau-U values range from -1.0 to 1.0, where a Tau-U value greater than zero indicates that intervention scores tend to be higher than baseline scores. Tau-U computation proceeded according to the steps laid out by Vannest and Ninci (2015): (a) the baseline trend level was calculated (Tau-U $_{\text{trend A}}$), and (b) if a baseline trend at or above 0.2 in the expected direction of the intervention was detected, the Tau-U coefficient that accounts for baseline trend (Tau-U $_{\text{A vs B + trend B – trend A}}$) was computed, otherwise, Tau-U without baseline correction (Tau-U $_{\text{A vs B + trend B}}$) was calculated. An omnibus Tau-U was obtained to measure the overall effect of the intervention. The decision to use either a fixed or random effects model to obtain the omnibus Tau-U was based on the heterogeneity of the data. A Tau-U value of 0.20 was interpreted as a small intervention effect, greater than 0.20 and smaller than 0.60 moderate, greater than 0.60 and less than 0.80 large, and over 0.80 very large (Vannest & Ninci, 2015). Finally, a piecewise regression analysis (level 1 analysis) for each participant and a hierarchical piecewise linear-regression analysis on the aggregated data (level 2 analysis) were conducted using the Huitema and McKean model (Huitema & McKean, 2000) to provide an inferential statistical validation of the results. Both level 1 and level 2 regression analysis were conducted using the SCAN package for R by Wilbert (2018).

Results

Figure 4 displays the students' number of problems solved correctly during the baseline and intervention conditions. Table 1 presents descriptive statistics on the performance of the students to solve subtraction problems.

Figure 4

Number of Subtraction Problems Solved Correctly for Aylin, Blanka, Carla, and Dana

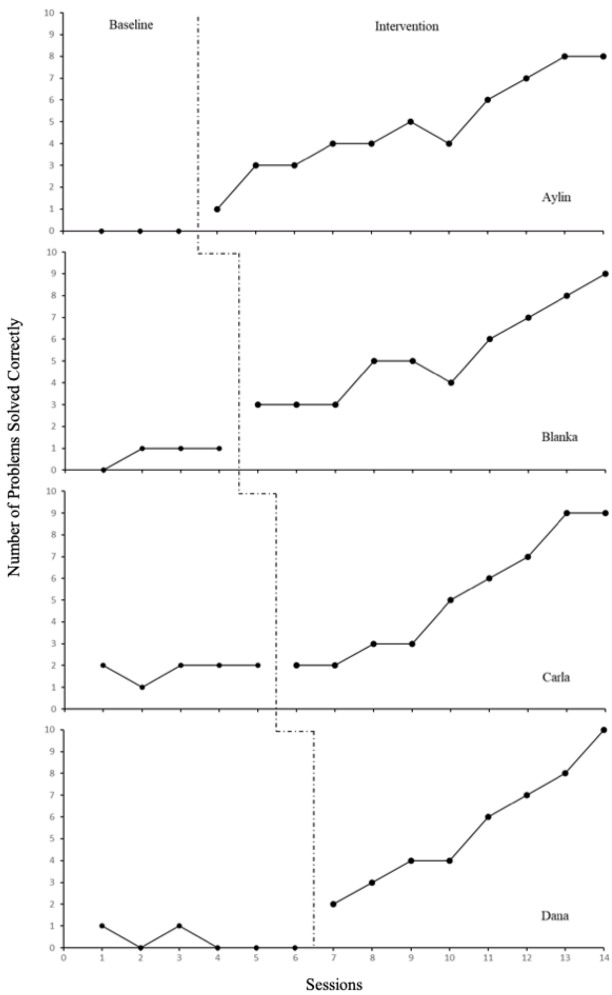


Table 1

Descriptive Statistics for Number of Subtraction Problems Solved Correctly

Student	*N*		*M (SD)*		*Md (IQR)*		Range	
	B	*I*	*B*	*I*	*B*	*I*	*B*	*I*
Aylin	3	11	0.00 (0.00)	4.82 (2.07)	0 (0.00)	4 (3.00)	0–0	1–8
Blanka	4	10	0.75 (0.50)	5.30 (2.16)	1 (0.25)	5 (3.50)	0–1	3–9
Carla	5	9	1.80 (0.45)	5.11 (2.80)	2 (0.00)	5 (4.00)	1–2	2–9
Dana	6	8	0.33 (0.52)	5.50 (2.73)	0 (0.75)	5 (3.50)	0-1	2–10

Note. B = Baseline, I = Intervention

Aylin. During the baseline phase, Aylin's performance was stable, she did not solve any of the subtraction problems correctly. In the intervention phase, Aylin demonstrated a stepwise improvement during the first six sessions with continuous improvement afterwards. Aylyn increased her performance from one subtraction problem solved correctly at the beginning of the intervention to eight by the last two days of the intervention. On average, Aylin solved 4.82 (range = 1 – 8) problems correctly during intervention.

Blanka. In the baseline phase, Blanka's performance was also low and exhibited a slight upward trend, with a mean of 0.75 (range = 0 – 1) problems solved correctly. Immediately after entering the intervention, her performance improved to three problems solved correctly within the first three intervention sessions, followed by a stepwise increase during the next two sessions, and then a steady growth in the last four intervention sessions. By the last intervention session, Blanka solved nine out of ten problems correctly. Blanka's mean performance during intervention was 5.30 (range = 3 – 9) problems solved correctly.

Carla. Carla solved mostly two problems correctly during the baseline phase. Her baseline performance was stable and averaged 1.80 (range = 1 – 2) problems correct. During intervention, Carla's performance started improving relative to baseline from the third session onwards, and continuously grew after the fifth intervention session. By the end of the intervention, Carla was able to solve nine problems correctly. On average, Carla solved 5.11 (range = 2 – 9) problems solved correctly during intervention.

Dana. In the baseline phase, Dana's performance exhibited a downward trend, she solved from zero to one problem correctly with a mean of 0.33 (range = 0 – 1). In the intervention phase, Dana increased her performance in a steady upward trend from two problems solved correctly just after the introduction of the intervention to ten by the end of the intervention, with a mean of 5.50 (range = 2 – 10) problems solved correctly.

The range of the effect size values suggests the touch point instructional package was effective to highly effective to improve the subtraction skills of elementary students at-risk for LD. In particular, PND across students ranged from 77.78% to 100%, with an overall PND of 94.45%. Tau-U effect sizes across the students ranged from 0.75 to 0.99 ($p < 0.001$), which are considered large to very large (Vannest & Ninci, 2015). Due to the lack of heterogeneity across the Tau-U effect sizes, a fixed effects model was applied to obtain an omnibus Tau-U. The overall Tau-U across students was 0.86 (CI95 = [0.66, 1.00], $p < 0.01$).

Table 2
Effect Sizes for Number of Subtraction Problems Solved Correctly

Student	PND	Tau-U [CI_{95}]
Aylin	100.00%	0.92** [0.52, 1.00]
Blanka	100.00%	0.81** [0.42, 1.00]
Carla	77.78%	0.75** [0.37, 1.00]
Dana	100.00%	0.99** [0.54, 1.00]
Omnibus	94.45%	0.86** [0.66, 1.00]

*Note. **p < 0.01*

Visual analysis indicated that baseline data was stable for two of the students, whereas one student displayed a slow decelerating trend and another a slight accelerating trend. An analysis of the students' baseline data determined none of the baseline trends were statistically significant. Thus, piecewise regression analysis of students' data and a hierarchical piecewise linear-regression analysis were conducted under the assumption of no baseline trend for all students. Due to the short duration of the baseline phases, this assumption theoretically may increase the risk of a beta error, which warrants a cautious interpretation of the results. As Table 3 illustrates, a statistically significant positive slope change estimate (Δ slope range = 0.65 – 1.10, $p < 0.001$) was found for all four students. On the other hand, a significant immediate change estimate with the introduction of the intervention was noted for three of the students (Δ level range = 1.33 – 1.59, $p < 0.05$), for one student the immediate change estimate was not significant (Δ level range = -0.69, $p = 0.15$). Moreover, visual analysis suggested that the performance growth of second language students was slower than the native language student, therefore, a hierarchical piecewise linear-regression analysis (level 2) was conducted to investigate the aggregated effect of the intervention and a potential interaction between intervention performance and second language learner status. Results showed a significant estimate for immediate change of 1.33 ($p < 0.05$) and a significant slope change estimate of 1.10 ($p < 0.001$) on the overall performance from baseline to intervention, however, no significant main effect for second language status (SLL = 0.58, $p = 0.44$) was observed. Furthermore, such analysis also revealed a significant slope change estimate (Δ Slope = -0.36, $p < 0.01$) from baseline to intervention between second language students and the native language student during intervention. This indicates that for second language students the performance slope during intervention was 0.36 slower than for the native language student. The overall immediate change estimate from baseline to the onset of the intervention for second language students was lower (Δ Level =-0.49, $p = 0.42$) than for the native language student, but this estimate was not statistically significant.

Table 3
Piecewise Regression Model for Number of Subtraction Problems Solved Correctly (Level 1 Analysis)

Student		ß	SE	t	Δ R^2
	Intercept	0.00	0.34	0.00	
Aylin	D Level[a]	1.59	0.48	3.35**	0.04
	D Slope[b]	0.65	0.06	11.50***	0.44
	Intercept	0.75	0.35	2.12	
Blanka	D Level[a]	1.52	0.55	2.79*	0.04
	D Slope[b]	0.67	0.08	8.63***	0.37
	Intercept	1.80	0.26	6.95***	
Carla	D Level[a]	-0.69	0.44	-1.57	0.01
	D Slope[b]	1.00	0.08	13.38***	0.61
	Intercept	0.33	0.21	1.58	
Dana	D Level[a]	1.33	0.40	3.37**	0.02
	D Slope[b]	1.10	0.08	13.70***	0.35

Note. a. Immediate change estimate from the baseline phase to the intervention phase.
b. Slope change estimate from the baseline phase to the intervention phase.
*Note. *p < 0.05, **p < 0.01, ***p < 0.001*

Table 4
Hierarchical Piecewise Regression Model for the Aggregated Student Data (Level 2 Analysis)

	ß	SE	df	t
Intercept	0.33	0.53	48	0.63
Δ Level[a]	1.33	0.53	48	2.49*
Δ Slope[b]	1.10	0.11	48	10.14***
sll	0.58	0.62	2	0.93
Δ Level[a]: SLL	-0.49	0.62	48	-0.8
Δ Slope[b]: SLL	-0.36	0.12	48	-3.05**

Note. SLL = Second language learner status.
Note. a. Immediate change estimate from the baseline phase to the intervention phase.
b. Slope change estimate from the baseline phase to the intervention phase.
*Note. *p < 0.05, **p < 0.01, ***p < 0.001*

Social Validity

End-of-intervention interview responses depicted positive students' attitudes towards the use of the touch points strategy. Aylin remarked, "I really liked working with the touch points. Even though I thought I would never learn how to subtract." She added, "The touch points made things very easy. I would like to use them in the future, too. Through working with the touch points, I lost my fear of math." Blanka explained, "Working with the sponge rubber digits was fun, and I really liked getting feedback on how well I did." However, when asked if she wanted to continue using touch points, she replied: "No, I don't need them anymore." Carla stated, "Being able to use the touch points made math very easy." She also voiced that she did not need the materials any longer: "I can do all the subtractions in my head now." She added, "Math is fun. I have not only become better, but also much faster." Dana said, "I liked the touch points, but not too much. Using my fingers is easier for me." However, she recognized, "I think that I can now do subtractions quicker and better." When asked if she would like to continue working with touch points, she answered, "No, I would rather play with other kids. I still don't like math very much."

Discussion

This replication study examined the effects of a touch point instructional package to foster the subtraction skills of four German elementary students at-risk for LD. Findings showed that the intervention was effective to very effective to enhance the ability of students to solve subtraction problems within 18 with two-digit minuends and one-digit subtrahends requiring crossing over the tens. All of the students were able to sustainably increase the number of correct responses using the touch point intervention from baseline (M = 0.72) to intervention (M = 5.18). Moreover, by the last two intervention sessions, students solved between 8 to 10 problems correctly as compared to between 0 to 2 problems during baseline. Students' performance improved during the course of the intervention as they learned and practiced the touch points strategy. Overall, both PND and Tau-U effect sizes at the individual and aggregated level were large to very large, which indicates the intervention was effective to very effective. Our findings are in alignment with previous research (Scott, 1993; Waters & Boon, 2011) that reported touch points instruction is effective to teach subtraction skills to students with disabilities.

Visual analysis in conjunction with a piecewise regression analysis indicated that the intervention did have a positive effect to improve the subtraction skills for all the students over time during the intervention. During intervention, there was an overall increase on the performance rate for all the students and an immediate increase in level from baseline to intervention for three of the students. It was noted that after the six instructional lessons, the performance across all the students continued improving in a steady manner. Thus, it is hypothesized that further independent practice upon completion of instruction helped the students to continue learning and internalizing the use of the strategy. In addition, hierarchical piecewise regression analysis results confirmed visual analysis that suggested that second language students' performance increased in a slower and more stepped fashion than for the native language student, this difference might have been due to some language struggles that these students might have had to overcome during instruction. However, this finding must be interpreted with caution as only one of the students was a native speaker. More interestingly, the second language students were able to catch up by the end of the intervention performing at the same level as the native language student. Thus, the hands-on and visual nature of the strategy along with direct instruction may have facilitated the acquisition of the steps to solve subtraction problems for all four participants. Furthermore, the results indicated that the use of touch points administered over a relatively short period, lasting from 8 to 11 sessions, yielded positive change on the performance across all the participants. This is consistent with previous results reported by Grünke et al. (2018), that found the same effects on learning single-digit addition skills of four German elementary students with intellectual and developmental disabilities. Therefore, findings from both studies suggests that providing a brief dosage of touch points instruction may be sufficient to effectively facilitate the learning of basic addition and subtraction skills for students with or at-risk for disabilities.

In terms of social validity, student responses to post-intervention interviews indicated that in general the touch points method was well-received by the students. Only one student, Dana, seemed not to be completely enthusiastic about using the touch points strategy. She stated the touch points procedure was more strenuous for her than finger counting. Unlike Dana, the other three students stated they enjoyed using the touch points strategy. Overall, by the completion of the study, all four students perceived an improvement of their subtraction skills, and felt the performance feedback provided during intervention motivated them to do better.

In summary, the findings of this study add to the growing body of literature on the effectiveness of the touch point strategy for students with disabilities. This investigation showed the touch point instructional package can be effective to improve the subtraction skills of first year German students at-risk for LD, some of which, were also second language learners.

Limitations

Several limitations should be considered in interpreting the results. First, the external validity and generalizability of the results is limited by the small number of participants. Further replications are needed to address this limitation. Second, participants were identified as at-risk for LD based on their academic performance during their first school year, and were selected according to the results of an informal mathematics assessment and an evaluation of their workbooks. Had standardized assessment data also been collected, it could have been determined whether the students met the criteria for an LD in Germany. Third, the intervention was conducted in a one-to-one format in a separate room. This instructional environment might have been conducive for strategy learning and thus boosting students' performance during intervention. Fourth, performance feedback and the use of rewards during the intervention may have contributed further to uplift students' motivation in learning the touch points strategy and performing better during intervention than in baseline. Even though these motivational factors could have been counter-balanced by the attention and encouragement delivered during the baseline phase, their influence on the students' intervention performance cannot be completely discarded. Fifth, repeated practice on the set of subtraction problems during the intervention sessions might have produced a facilitative effect that resulted in an overall increase of students' performance by the end of the intervention condition. Moreover, error correction procedures, performance feedback and repeated exposure to the set of problems during intervention might have promoted rote memorization of the answers. Sixth, procedural reliability was self-collected by the interventionists. Due to interventionist bias, this method tends to inflate reliability ratings, thus weakening internal validity (Lane et al., 2009). In this study, however, the extensive interventionists' training and the use of detailed procedural scripts might have led to a reduction of the interventionists' bias and likely promoted higher procedural adherence and accuracy (Fallon, 2018; King-Sears et al., 2018). Lastly, follow-up and maintenance data were not collected, thus, the short and long-term effects of the intervention are unknown.

Implications for the Classroom and Future Research

The findings of this study provide evidence that a touch point instructional package has the potential to enrich learning of subtraction skills for students at-risk for LD. However, to implement the method in practice, it is necessary that the teachers use differentiated instruction and adapt the instructional materials according to the competence level of each student. Additionally, because individual instruction in the schools is available in exceptional cases, future studies should explore the effectiveness of touch points instruction delivered in small groups or peer-tutoring formats in classroom settings. A peer-tutoring implementation of the touch point intervention seems to be promising as the strategy has a systematic approach that can easily be learned and conveyed by peer tutors. Future studies are warranted to evaluate whether touch point interventions are as effective to teach multiplication and division skills as well, and other math life skills such as money and time management. In addition, studies should also explore the effectiveness of the touch points method contrasted to other methods (e.g., number line) that enable students to expand their basic mathematics skills. Finally, future research should also investigate the effectiveness of computer-based instruction of the touch point strategy, such as TouchMath PRO, and other applications available within the program.

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