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Errors Made by Elementary Fourth Grade Students When Modelling Word Problems and the Elimination of Those Errors through Scaffolding

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Abstract

This study aims to identify errors made by primary school students when modelling word problems and to eliminate those errors through scaffolding. A 10-question problem-solving achievement test was used in the research. The qualitative and quantitative designs were utilized together. The study group of the quantitative design comprises 248 elementary 4th grade students attending nine classes at three state schools in the city centre of Kütahya, chosen with the cluster sampling method. Frequency analysis and discriminant analysis were performed to analyse the quantitative data. The qualitative data were collected through clinical interviewing. The study group with whom the clinical interviews were performed comprises 30 primary school students in the class closest to the average problem-solving achievement among the nine classes. As a result, it was observed that most of the errors made by the students were caused by the use of the number operator model, which was followed by incorrect relations, number consideration, missing critical information, an inability to determine structure and relation and incorrect diagrams. The discriminant analysis shows that the biggest contribution to discriminating between students with high and low levels of modelling achievement is made by errors originating from using the number operator model, and this type of error is followed by incorrect relations, an inability to determine structure and relation and number consideration models respectively. It was concluded that errors originating from missing critical information are mostly made by successful students and the ratio of errors originating from incorrect diagrams does not affect the distinction between successful and unsuccessful students. The research also found that the modelling cycle of students does not benefit from the interpretation and validation stages. Finally, it was seen that more than half of errors made during modelling can be corrected through scaffolding.

Keywords: Word problems, modelling cycle, Error analysis and scaffolding.

Introduction

The PISA survey has become an influential factor in reforming educational practices (Liang, 2010) and making decisions about educational policy (Yore, Anderson, & Hung Chiu, 2010). PISA results showed that the competencies measured in PISA surveys are better predictors for 15 year-old students' later success (Schleicher, 2007). One of the skills measured in PISA is mathematical literacy, which can be defined as "turning real-life

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problems into mathematics" and "interpreting existing knowledge and adapting it to real-life" (Blum & Niss, 1991; Lesh & Doerr, 2003). PISA categorizes problem solvers into seven levels. Those under the first level are the students who cannot solve problems, whereas the first level defines the students who can solve routine problems when the question is expressed clearly and all the required information is provided for solution. The second category defines the students who can reason simple relations that are evident at the first glance. The third level and above express the students who can adapt mathematics to real-life situations. PISA (2015) revealed that 45.9% of the high school students trained in OECD countries are below the third level. These results show that approximately half of the high school students in OECD countries have trouble in solving real life problems. This problem was also observed during elementary school years in some studies (e.g., Verschaffel, Greer & De Corte, 2000; Verschaffel, De Corte & Vierstraete, 1999; Xin, Lin, Zhang & Yan, 2007).

Turkish Ministry of National Education (2005) emphasized problem solving, training of problem solving strategy and skills of modeling sense-making problems in mathematics curriculum from the first years of elementary school. However, despite a strong emphasis in the curriculum, PISA (2015) revealed that 77.6% of Turkish high school students cannot solve sense-making problems. In their studies, Anderson (2010), Grimm (2008), Jordan, Kaplan and Hanich (2002) stated that it becomes harder to furnish students with problem solving skills at later ages if these skills haven't been acquired at early ages. On the other hand, Wischgoll, Christine and Reusser (2015) advocated the idea that "Errors are indicators of learners' misunderstanding. While learners are making errors, the gaps in their understanding become apparent, and learners gain understanding by bridging these gaps. Wischgoll and others (2015) regards errors as opportunities rather than disadvantages and suggests that knowing the obstacles before skill development will contribute to the studies towards skill development. In this context, it has been decided to conduct a study to determine the errors in problem solving process and to conduct the study on elementary students since problem solving is a skill that should be acquired at early ages. Afterwards, the problem type to be used in this study has been determined.

In literature, problems are divided into two: routine (exercise-type) and non-routine problems (sense-making problems) while non-routine problems are again divided into two: those with closed-ended answer and those with open-ended answer (Akay, Soybaş & Argün, 2006; Foong, 2002). According to Altun (2007), the question *"Ali buys 2 pencils, one at 3 TL, how much TL does he pay?"* is a routine while the question *"One pays 3 TL to divide an iron bar into two, how much TL is paid to divide the iron bar into four?"* is a closed-ended non-routine problem. While it is simply enough to form 3x2 equation for solution in the former problem, thinking that 3 cuts are required to divide the iron bar into four, 3x3 equation should be formed in the latter, which demands real life knowledge to solve non-routine problems. Whereas the iron bar question is close-ended since it has one answer, the question "*A school with 325 students wants to take its students on a picnic by buses with 50 seats each. How many buses are needed to take all the students on a picnic*?" is an open-ended non-routine problem. For solution, "325/5= 6.5"is not enough; a mathematical solution has no chance to occur in real life conditions. In this context, since the number of buses cannot be expressed as 0.5, how 25 students will go on a picnic should be fictionalized. Suggestions for answers here might vary from one student to another like "*Let's allocate the remaining students to the other buses*" or "*We can take a smaller minibus.*"

In some studies (e.g., Clarkson, 1991; Clements, 1982; Clements & Ellerton, 1996; Marinas & Clements, 1990; Singh et al*.,* 2010; Singhatat, 1991) determined the errors in routine problems. In some studies (e.g., Verschaffel et al., 2000; Verschaffel et al., 1999;

Xin et al., 2007) determined the errors in open-ended non-routine problems. In a study by Yeo (2004) in closed-ended non-routine problems, a limited number of questions (3 questions) and limited amount of sampling (56 students) were used and the rates of generally made errors weren't stated. No other study was determined in Turkey except the one by Ulu, Tertemiz and Peker (2016a) to determine the source of errors elementary students make in non-routine problems. In this context, it was decided to determine the errors in closed-ended non-routine problems in this study. It was thought that it would be effective in identifying the errors made by students with lower problem-solving achievement compared to students with higher problem-solving achievement when identifying the type of support needed. Newman (1977) developed an inventory to classify the errors made during the problem-solving process and word problems have generally been analyzed together with this inventory (Singh, Rahman & Hoon, 2010; Clements & Ellerton, 1996; Clarkson, 1991). Hong (1993) developed an inventory for identifying errors originating from modeling during the problem-solving process (mental model); it was deemed more suitable to use Hong's inventory since the purpose was to identify the errors made during modeling.

It was thought in PISA (2015) that knowing what successful countries do differently in mathematical literacy compared to those with less success would contribute to decreasing the errors. In mathematical literacy, the country with the highest success is Singapore. Kaur (2001) suggested that Singapore's success in problem solving was thanks to their choices in both mathematics curriculum and problem selection. With structural reforms in the mathematics curriculum in Singapore in 1992, the time spent on mathematical content where factual and operational skills are prominent was decreased 30% and problem solving skills were added at the same percentage (Kaur, 2001). Singapore mathematics curriculum gives as much importance to the time spent on problem solving activities as the problem types to be solved in the lesson; in this country, the subject starts to be taught with routine problems and non-routine problems follow when the subject has been understood (Kaur & Yeap, 2009).

Another feature of Singapore mathematics curriculum is that it is individualistic; it gives every student to advance with his/her own pace and another subject doesn't start until the former has been mastered. In this system, teachers use scaffolding method. According to Wischgoll, Christine and Reusser (2015) the concept of scaffolding as well as the concept of the zone of proximal development ascribes importance to the learner interacting with someone more capable than themselves during problem solving. According to Wischgoll and others (2015) scaffolding is described as the support given by the instructor to the learner so that he/she may understand the problem situation, aiming through this support to transfer the responsibility from instructor to learner. The idea which prevails here is that the support given should help to identify areas in which students are struggling to understand the problem, or think that they understand the problem (even if incorrectly), and clearing those points up. During the process, the support provider is called the "tutor", while the support recipient is called the "tutee." In a study by Wischgoll and others (2015), it was seen that this system is effective in correcting student errors instantly, but no such study has been conducted in Turkey. This method was utilized in the study because it aimed to correct the areas in which students were making errors through local interventions. To this end, this study seeks to answer the following questions:

1) How are the errors made by students when modeling word problems distributed according to error type?
2) What is the relative order of importance of each error type in classifying students with higher and lower problem-solving achievement?

3) Is scaffolding effective in eliminating errors made during the modeling process

Theoretical framework

According to Hegarty, Mayer and Monk (1995), there are two approaches in problem solving process: key word and comprehension- oriented. They stated that individuals decide their operations upon certain special words (more, less, times, etc.) they choose without understanding the problem in key word- oriented solution approach whereas in comprehension-oriented approach, they decide their operations within the context of the characters, time, place and relation between the events in the problem. In some studies, (e.g., Hegarty et al., 1995; Viennot & Moreau 2007; Soylu & Soylu, 2006; Pape, 2004) revealed that students with lower problem solving success do over-regularization whereas successful students achieve more real solutions by forming accurate relations between the problem elements (characters, time, place and events).

The short-cut approach is defined as how students use readily available solutions in their memories when they confront a problem that is like several problems they have solved before (Jitendra & Hoff, 1996; Steele & Johanning, 2004; Viennot & Moreau, 2007). This theory assumes that solving different kinds of questions will enhance the problem-solving achievement as it will increase the number of solution methods stored in the memory. Yet, when the question is changed a little bit or students confront with a new question, this approach may fail to solve the problem (Viennot & Moreau, 2007). The readily available solutions mentioned in the short-cut approach are addressed within the scope of procedural knowledge (Anderson, 2010; Brynes & Wasik, 1991; Baroody, Feil, & Johnson, 2007).

In a study by Soylu and Soylu (2006), most of the students' answers to the question "*When Ali gives 5 of his apples to Ayşe, he has 10 apples left, so how many apples did Ali have in the beginning?"* was 10 − 5 = 5. It was determined through an interview that as a result of over-regularizing the expression *"left*", students did subtraction instead of addition. In a study by Viennot and Moreau (2007), students were first asked "*Luke comes to school with 15 marbles. He plays two games and loses 7 marbles in the first game. At the end of the second game, Luke, being a good gamer, ends up with 34 marbles. How many marbles did Luke lose in the second game if he had lost or how many did he win if he had won the second game?"* and then the same problem was asked again by replacing *"a good gamer"* with *"a bad gamer"*. At the end of the study, it was determined that some of the students who wrote "*He won 26 marbles*" to the questions with *"a good gamer"* changed answer and said "*He lost 26 marbles*" to the questions with *"a bad gamer"*. These two cases show that word-oriented solutions prevent achieving realistic solutions and so one should go for comprehension-oriented solutions.

In some studies (e.g., Anderson, 2010; Brynes & Wasik, 1991; Baroody *et al*., 2007) was observed that the procedural knowledge may fall insufficient in the first-time situations and contextual knowledge is needed for such situations. Kieren (1993), Baroody and others (2007) defined conceptual knowledge as associating recently-learned knowledge with previously-learned knowledge and real life, constructing the knowledge in accordance with individual traits and processing it through a rational sieve. They emphasized that procedural knowledge may be enough for solving routine problems but process skills and conceptual knowledge are in the forefront as non-routine problems have a more complex structure. According to NTCM (2010) and MEB (2005), today's educational programs advocate the idea "conceptual understanding rather than procedural knowledge or rule-driven computation" both mathematics-wise in general and problem solving-wise.

Emphasizing that there are similarities between understanding a story and a problem, they stated that just as students should focus on the whole text to understand the story, so should they focus on the whole problem text instead of focusing on a single word to understand the problem accurately (e.g. Dijk & Knitsch, 1983; Kintsch & Greeno, 1985; Reusser, 1985; Staub & Reusser, 1995). Staub and Reusser (1995) carried this idea one step further and structured a model called "from the text to situation and from the situation to the equation" that aims to help students solve a problem just as they analyze a story. This model was initially used to prevent elementary school students from over-regularizing while solving problems requiring addition and subtraction; however, it became the origin of modeling cycle- developed by Blum and Leiss (2006) and finalized by Borromeo Ferri (2006)- aiming at solving realistic problems. The modeling cycle developed by Borromeo Ferri (2006:92) is presented in Figure 1.

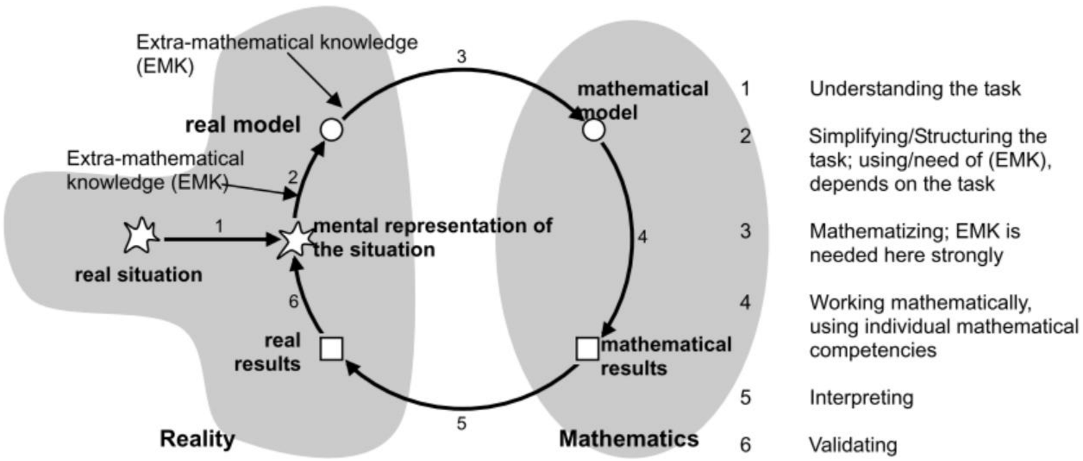


Figure 1. Modeling cycle from Borromeo Ferri (2006:92)

According to Figure 1, in the first phase, the mental representation of the situation model should be established in the real situation and it is expected from the solver to understand the problem. Contemporary approaches to story problem solving have emphasized the conceptual understanding of a story problem before attempting any solution that involves selecting and applying an arithmetic operation for solution (Jonassen, 2003). According to Kintsch (1988), in conceptual understanding phase, an individual needs to comprehend the problem text literally, inferentially. Literal comprehension, the first level of comprehension, requires that a student can extract information that is explicitly stated in a passage (Carnine et al., 2010; Lapp & Flood, 1983; McCormick, 1992). According to Kintsch, literal comprehension lets us find answers for WH-questions (e.g. who, what, where, when, how) in the text. This level of understanding is dependent upon students' word-level processing skills, or their ability to accurately identify individual words and understand the meaning created by the combination of words into propositions and sentences (Perfetti, Landi, & Oakhill, 2005).

Inferential comprehension aims to establish empathy between the character in the text and the reader and to determine why the event in the text is being told, what its effects are on the reader, what the motives of the main character in the text are, what the main idea the author is trying to convey in the text is and the cause and effect relations between the events (Kneene & Zimmermann, 1997). Kintsch (1988) expresses inferential comprehension as a situational model and states that establishing a situational model during comprehension will activate the background information of the reader about the event and thus richer information units will be reached which are inclined to real life and whose connection with the background information has been established. It is stated that the main purpose in reading a text is thought to be inferential comprehension but literal comprehension is a prerequisite for inferential comprehension to occur (Allen, 1985; Kinsch, 1988; Suk, 1997; Vacca, Vacca, Gove, Burkey, Lenhart & McKeon, 2006; Ulu, 2016; Özsoy, Kuruyer & Çakıroğlu, 2015).

In some studies (e.g., Kispal, 2008; Chikalanga, 1992; Zwiers, 2004; Presley, 2000; Kintsch, 1988) stated that an individual who made an inference during comprehension was at the same time reasoning. The role of reasoning during problem solving was defined as reaching a solution by integrating every proposition in the problem text in a logical consistency (Leighton & Sternberg, 2004). Regarding the definitions above, significant resemblances are seen between reading comprehension and reasoning skill during problem solving. Background information should be activated other information should be reached about the explicit information in the text both in inferential comprehension during reading comprehension and in reasoning during problem solving. Literature shows that a positive relation exists between problem solving and reasoning skills (Barbey & Barsalou, 2009; Çelik & Özdemir, 2011; Çetin & Ertekin, 2011; Umay, 2003; Yurt & Sünbül, 2014). In some studies (e.g., Clarkson, 1991; Clements, 1982; Clements & Ellerton, 1996; Marinas and Clements, 1990; Singh et al., 2010; Singhatat, 1991) found that the errors in non-routine problems were mostly due to lack of understanding. Staub and Reusser (1995) stated that the comprehension strategies can be utilized in this phase, and in the studies performed by Hite (2009) and Ulu, Tertemiz and Peker (2016b), it was seen that the problem-solving achievement can be improved just by giving training for comprehension strategies.

As seen in Figure 1, individuals who envision the problem situation and make sense of it are expected to simplify and structure the problem and those who have achieved this stage will have formed the real model. Reusser (1995), Niss, (2003), Borromeo Ferri (2007), Blum and Borromeo Ferri (2009) define this stage the process of forming new problems by collecting the necessary new information according to hidden actions revealed in comprehension process. Simplifying is the discrimination of information necessary and unnecessary for solution. Structuring is the process of associating the problem elements based on the scenario in the problem text (Reusser, 1995; Niss, 2003). Borromeo Ferri (2006) thinks that the structuring step is a process of internal representation, in other words, taking the mental picture of the problem situation. Internal representation is composed of two steps: recall and association. Charles and Lester (1984), and Altun (2007) define individuals' asking themselves “Have I solved a similar problem before” as recall. Organizing the information from the long-term memory, discriminating what is given in the problem in accordance with what is asked and adapting it into the current problem situation are defined as association (Steele & Johanning, 2004; Cummins, Kintsch, Reusser & Weimer, 1988).

During the third stage, the solver is supposed to build the mathematical model with reference to the real model and in this process, the individual is supposed to do mathematization. Mathematization is a process in which a problem already existing in the brain structured non-graphically is converted into tables, figures or symbols (Borromeo Ferri & Blum, 2009; Borromeo Ferri, 2006). According to Niss (2003), at this stage, the strategy to be used for solution is operated and the next stage- mathematical working- follows. In mathematical working process, figures, tables or equations are put into service to reach a mathematical solution. In some studies (e.g., Clarkson, 1991; Clements, 1982; Clements & Ellerton, 1996; Marinas and Clements, 1990; Singh et al., 2010; Singhatat, 1991) found that the second most frequent error type of elementary students in routine problems is transformation-based errors.

According to Figure 1, the last stage of the process is turning back from mathematical results to real results and within this process, the individual is supposed to interpret the mathematical result s/he has ended up with. Blum (2015) considers the interpretation stage as questioning the probability of the resultant mathematical outcome in real life. The individual who has dealt with the interpretation stage will now do validation and if s/he feels that the outcome is inconsistent with real life, turning back to the real model, s/he will try to sort out these inconsistencies whereas if s/he doesn't see any inconsistencies and becomes satisfied with the outcome, s/he will report the final result and turn back to the real situation. According to Borromeo Ferri (2005), it is wrong to consider the validation process as checking the accuracy of the mathematical operations; rather, validation process is when the individual questions whether his/her solution and resultant answer are reasonable according to real-life conditions. It was explored in the studies conducted by Maass (2007), Wijaya and others (2014), Eraslan and Kant (2015) that the students gave up solving the problem generally in the mathematical result step and ignored two most important steps of the modeling cycle: interpretation and validation. The studies conducted by Teong (2000), Özsoy and Ataman (2009) found that using the control processes reduced the errors.

Method

The research model

Present study was conducted based on mixed methods design (Tashakkori & Teddlie, 2010). Mixed methods research merges qualitative and quantitative data to answer the research question (Creswell, 2014). There are other terms used to refer to mixed methods such as integration, synthesis, qualitative and quantitative methods, multiple methods, and mixed methodology (Byrman, 2006; Tashakkori & Teddlie, 2010). Sequential explanatory design of mixed models was used in the research. This design can be defined as supporting the process of quantitative data collection and analysis with qualitative data collection and analysis. In sequential explanatory design, the research mostly focuses on quantitative data. Qualitative and quantitative data are integrated within the interpretation process of the research because the aim of this design is to support interpretation and explanation of quantitative data with qualitative data (Creswell, 2014).

Quantitative Strand

The questions "How are the errors made by students when modeling word problems distributed according to error type?" and "What is the relative order of importance of each error type in classifying students with higher and lower problem-solving achievement?" were analyzed using the survey method. The survey model aims to reveal the situation as it currently is and the researcher cannot have any manipulative influence in this model (Karasar, 2002).

Samples

The cluster sampling method was used for the quantitative strand. Karasar (2002) suggests that when all the elements in the population don't have the chance to be chosen one by one, choosing is to be done among the whole group using cluster sampling. In cluster sampling, the chance to be chosen isn't valid for the elements alone but for the whole group with its elements. In this context, while determining the study group of the research, not choosing the individuals but choosing the classes was the case. First, the schools were divided into three groups (high, moderate, and low) based on TEOG (Transition from Primary to Secondary Education) exam scores, with a school of each group being chosen using the unbiased selection method. 248 fourth-grade students attending the nine classes of the chosen primary schools were set the problem-solving achievement test. The sample is composed of 138 female and 110 male students. The ages of the students vary between ten years and three months and ten years and eleven months old.

Data Collection Tool

First of all, a problem-solving achievement test was developed to classify the errors made by the students in the research. The problem-solving achievement test is composed of 10 word problems used in the studies performed by Ulu and others (2016a), Altun (2007), Yazgan and Bintaş (2005), Griffin and Jitendra (2008). While developing the test, expert opinion of three experts having completed their PhD in mathematics education in elementary teaching. The experts decided that the test had better be comprised of questions that were appropriate for using problem solving strategies suggested by MEB (2005). Table 1 shows the strategies that could be used in solving the questions in this test.

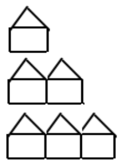
Table 1. *Strategies that could be used in solving the questions in problem solving test*

Questions	Writing a mathematical sentence	Drawing a diagram	Work backwards	Guess and Checking	Logical reasoning	Eliminating	Systematic listing
1	x	x					
2	x	x					x
3	x		x				x
4	x	x	x				x
5	x			x			x
6	x			x			
7	x			x		x	
8	x			x		x	
9	x	x			x		
10	x	x			x		x

The study to assess the validity and reliability of the scale was performed on 124 fourth-grade students at the school with the closest score to the Kütahya average based on the 2014/2015 YEP (Placement Scores). Firstly, the item difficulty and item distinctiveness of each question and secondly, the reliability coefficient (KR-20) of the scale was calculated in order to determine its validity and reliability. According Tekin (1997), items with an item difficulty index between 0 and 1 and difficulty indices between 0.30 and 0.70 are of a moderate difficulty level. The item difficulty indices of items in the scale vary between 0.32 and 0.48, which indicates that all of the problems in the test are of a moderate difficulty. The distinctiveness index varies between -1 and +1, with a value of 0.40 or higher demonstrating the distinctiveness of the items (Tekin, 1997). The distinctiveness indices of items in the test vary between 0.43 and 0.64, which indicates that all of the items are distinctive. The KR-20 value for the internal consistency of the scale was calculated as 0.84. If the KR-20 value is 0.70 or higher, it shows that the test has a high level of internal consistency and, therefore, reliability (Büyüköztürk, 2006).

According to Şekercioğlu, Bayat, and Bakır (2014), factor analysis of the scales scored as 0-1 should be conducted on tetrachoric correlation matrice. Because problem solving scale is scored as 0-1, construct validity (factor analysis) of the scale was done on tetrachoric correlation matrice. According to the analysis result, the fact that KMO value was .898 shows that the scale has sufficient sampling size for factor analysis and Barlett test results (X^2_{45}=881.338; p<.01) show that the variables have equal variance (Büyüköztürk, 2006). As a result of analysis, factor loads of the scale items varied between .898 and .496 and since factor loads were sufficient, it was decided to keep all the items in the scale (Büyüköztürk, 2006). It was also seen that with its one-dimension structure, the scale explains 66.32% of problem solving variance. Also confirmatory factor analysis (CFA) was performed to determine the validity of the scale based on the average scores, and it was seen that fit indices of the model established with the scale's one-factor structure (χ^2/sd=1.144, RMSEA=0.023, TLI=0.99, IFI=0.99, GFI=0.97) are sufficient. The problem-solving achievement test is shown in Table2.

Table 2. *Problem-solving achievement test*

1. Students in a classroom form a single line in the Physical Education class. Aslı is the third from first; Onur is in the centre. Now that there are 10 students between Aslı and Onur, how many students are there in the line?
2. A ball dropped from high bounces a distance of half of the height it has been dropped from. If the ball bounces 5 metres on the fourth bounce, from what height has it been dropped?
3. Rabbits reproduce at an astonishing pace. The population of rabbits doubles each year. If there are 3,200 rabbits in the forest after seven years, how many rabbits were there in the forest in the first year?
4. Trees will be planted at intervals of 5 metres on both sides of a 40-metre-long road. How many trees will it take to plant along the road?
5. In a farm with chickens and rabbits, there are 12 heads and 30 feet. How many rabbits are there in the farm?
6. I counted 10 bicycles and tricycles and 26 tyres passing through my house in the morning. How many of them are tricycles?
7. Eren is four years old and his father is 37 years old. How many years later will the father be four times older than Eren?
8. There are 24 men and nine women at a party. How many married couples will it take to make the number of men twice the number of women?
9. A man walks three steps forward, one step back. If he takes 56 steps this way, how many steps will he be away from the starting point?
10. Dilek is building a house of matchsticks. She needs six matchsticks to build a house. She needs 11 matchsticks to build two adjacent houses. She needs 16 matchsticks to build three adjacent houses. How many matchsticks will she need to build ten adjacent houses? 

Qualitative Strand

The answer to the question "Is scaffolding effective in eliminating errors made during the modeling process?" was found using the clinical interview method. Piaget argues that errors made by children provide important information on the way they think and that it is necessary to use the clinical interview method(a method using flexible questions) to explore the richness of students' thoughts and evaluate their cognitive skills (in Baki et al., 2002, p.5). According to Frederiksen, Glaser, Lesgold and Shafto (1990), standard tests can only determine the extent to which students can solve problems correctly or incorrectly, but do not question why they do it correctly or evaluate the methods used to achieve the right result. Karataş and Güven (2004) regard the clinical interview method as one of the measurement methods for evaluating problem-solving skills and think that the reasons behind errors made by students when solving problems can be revealed as they are in the process of solving them. Hunting (1997) stated that the clinical interview method is dynamic and allow the student him/herself to identify their errors.

Sample

While determining the study group out of whom qualitative data would be gathered, typical case sampling of purposeful sampling types was used. The basic aim in typical case sampling is to determine an average group to resemble the population (Yıldırım & Şimşek, 2008). The study group undergoing clinical interviews was made up of a class chosen from among the nine classes in the sample. The reason for which this group was chosen is that it was the closest to the average score out of all nine classes. The clinical interview study group is composed of total 30 students; 16 female and 14 male.

Data Collection

A clinical interview form was integrated with a version of the problem-solving error analysis inventory developed by Karataş and Güven (2003), Karataş and Güven (2004) and Newman (1977) and was transformed into an interview form to be used in identifying and eliminating errors made by the students during modeling. The form comprises the following general questions:

1) *What is given in the problem?*
2) *What is the problem asking of you?*
3) *Could you explain why you used each operation to solve the problem?*
4) *Do you think the result is correct?*
5) *If you think it is not, where may you have made the error?*
6) *Leading questions for incorrect solutions to direct the student to the correct solution.*

Since 40 minutes was enough for solving the problems during the pilot application of the problem-solving scale, this duration was taken as reference point in its real application. The problem-solving achievement test was primarily applied to 248 primary fourth-grade students. Correct and incorrect solutions and solutions left blank were identified and the achievement average of the test, as well as that of each class, was found. Clinical interviews were conducted with the class with the closest mean score (m=1.80) to the achievement mean score of the test (m=1.80). In the interviews, the students were asked to solve the problem vocally and explain why they were performing each operation while solving it. They were then expected to notice their errors and correct them through leading questions. The videos recorded, which involved 192 incorrect solutions, were watched by three mathematical branch experts and the errors were classified using Hong's (1993) error analysis inventory. The error analysis inventory is presented in Table 3.

Table 3. *Types of error*

	Sample behaviours
Missing critical information	The student understands most of the real situation, chooses the right strategy for the solution and works through that strategy correctly. He/she misses an action due to a lack of attention when transforming it into the situation model, therefore missing the internal and external representations. Since the representation step is missing, the mathematical operations he/she has performed are missing also.
Incorrect relations	Individuals perform internal representation during the transition from the real model to the mathematical model. During this stage, an incorrect relation between the problem elements (e.g., events, what is being asked) causes an incorrect external representation that enables the mathematical model. When the external representation is incorrect, incorrect operational choices occur.

Table 3 (Cont.). *Types of error*

	Sample behaviours
Inability to determine structure and relation	The student transfers a part of the real situation to the situation model, transforming that part into the real model and performing the correct mathematical solution. Yet, he/she cannot mentally structure other necessary operations and therefore quits solving the problem.
Incorrect diagrams	The student tries to solve the problem by drawing a diagram, but structures it incorrectly. This type of error originates from inability to understand the real situation or inability to perform the external representation despite understanding it.
Number consideration	The student tries to solve the problem by using prediction and control strategies. Yet, he/she works the mathematical model regardless of all the conditions in the real situation. He/she works the prediction and control processes by considering only the one side of the equation.
Number operator	The student orientates toward the mathematical model without understanding the real situation first, in other words, without establishing the situation model and real model of the problem.

Data analysis of quantitative and qualitative strands

The process is explained under one title since qualitative and quantitative data are analyzed together in this section. The content analysis (Elo & Kyngäs, 2008; Vaismoradi, Turunen & Bondas, 2013) technique was used to analyze the qualitative research data. Qualitative content analysis can be defined as the procedure of classifying and interpreting the content of written texts via encoding and creating themes or patterns systematically (Hsieh & Shannon, 2005). So, the clinical interviews performed with 30 students on problem solving test were first examined by three experts specialized in mathematics education in classroom teaching and errors in each question were coded. Next, it was decided that the codes were to be examined in six themes according to Hong's (1993) error analysis inventory. Finally, the themes and codes were evaluated in accordance with the modeling cycle stages developed by Borromeo Ferri (2006).

The clinical interviews were classified using the error analysis inventory, while the Kappa coefficients were looked up to determine correlation between the experts. The data obtained from the Kappa coefficient are interpreted as "Weak Correlation=< 0.20; Acceptable Correlation = 0.20-0.40; Moderate Correlation = 0.40-.60; Good Correlation = 0.60-0.80; Very Good Correlation = 0.80-1.00" (Şencan, 2005, p. 485). The kappa coefficients were found to be .83 for missing critical information, .85 for incorrect relations, .87 for mental incapacity, .97 for incorrect diagrams, .91 for mental incapacity, .82 for number consideration and .92 for number operator model. These values showed that the experts exhibit very good correlation in the classification of errors.

After the 192 incorrect solutions observed in the clinical interview classes had been classified by the experts, 1415 (i.e., 1607-192) errors observed in the remaining eight classes started to be classified. When classifying these 1415 errors, no separate clinical interview was performed; however, the classification was performed with regard to the opinions received from the 192 clinical interviews. Since there was very good correlation between the experts, only one mathematical expert classified the errors at this stage. Next, the ratios of error were calculated, first for each question, then in total. To explain the ratios of error made in each question using one example, given that errors originating from missing critical information were observed in the first question 28 times, the ratio of this error in the first question was calculated to be 11.29%. The ratios of errors made in total were calculated by adding the types of error classified for each question and dividing the sum by the expected number of solutions in the test (10x248=2480). To explain this process using an example, given that the errors originating from missing critical information are observed over all ten questions 300 times, the ratio of this error in the test as a whole will be calculated as 12.09% (=300/2480).

In the last stage, the errors were examined with regard to how they discriminated students with high and low problem-solving achievement. To this end, a discriminant analysis was performed. The discriminant analysis is a technique used to classify individuals or units, test theories as to whether individuals or units can be classified by predictions, investigate the differences between groups, evaluate the relative order of importance of independent variables in the classification using dependent variables and discriminate the least important, or unimportant, variables when classifying the groups (Büyüköztürk & Çokluk, 2008). When classifying students with high and low problem-solving achievement, the mean score of the test was found (m=1.81) and the students whose scores were above the arithmetic average were classified as having high problem-solving achievement, while the students whose scores were below the arithmetic average were classified as having low problem-solving achievement.

Findings

In this section, an answer was sought for the first problem question in the study: "How are the errors made by students when modeling word problems distributed according to error type?" In this context, the distribution of the errors made by the primary school students is presented in Table 4.

Table 4. *Frequency analysis of the errors in problem-solving achievement test*

Answers	*f*	%
Blank	424	17.10
Missing critical information	200	8.06
Incorrect relations	323	13.02
Incorrect diagrams	187	7.54
Mental incapacity	189	7.62
Number consideration model	256	10.32
Number operator model	458	18.22
Correct	448	18.10
Total	2480	100

According to Table 3, it can be observed that the ratio of solutions left blank over the test as a whole is 17.10%, the ratio of correct answers is 18.10% and the ratio of incorrect solutions is 64.80%. It is understood that most of the errors made by the students originated from the number operator model, with a ratio of 18.22%. This type of error is followed by incorrect relations at 13.02%, the number consideration model at 10.32%, missing critical information at 8.06%, mental incapacity at 7.62% and incorrect diagrams at 7.54%. Within this scope, an answer was sought to the second problem question in the study: "What is the relative order of importance of each error type in classifying students with higher and lower problem-solving achievement?" The findings obtained are presented in Table 5.

Table 5. *Wilks lambda test for group equality and standardized discriminant coefficients*

	Problem-solving Achievement	Ratio of Error	Wilks Lambda	Standardized Discriminant Coefficients	*F*	*p*
Missing critical information	Low	.61	.845	-.270	44.970	.000
	High	1.37				
Incorrect relations	Low	1.61	.883	.485	32.583	.000
	High	.90				
Incorrect diagrams	Low	.72	.993	.154	1.734	.189
	High	.88				
Mental incapacity	Low	1.00	.914	.457	23.075	.000
	High	.49				
Number consideration model	Low	1.20	.931	.410	18.159	.000
	High	.65				
Number operator model	Low	2.42	.795	.684	63.345	.000
	High	.54				

$p<.01$

According to Table5, with the exception of errors originating from incorrect diagrams ($p>.01$), the rest of the errors were effective in discriminating students with high and low problem-solving achievement ($p<.01$). As for the standardized discriminant coefficients (*d.c*), the order of importance of the errors in discriminating students with high and low problem-solving achievement is number operator model (*d.c*=.684), incorrect relations (*d.c*=485), mental incapacity (*d.c*=.457) and number consideration model (*d.c*=.410). It was found that errors originating from missing critical information (*d.c*=.-270) had a negative impact on the discrimination of errors because this type of error was made more frequently by students with high problem-solving achievement. The ratios of errors originating from incorrect diagrams were similar for both the low achievement group and the high achievement group. In the next stage, the contribution made by each error in classifying the groups with high and low problem-solving achievement was examined, and the findings obtained are presented in Table 6.

Table 6. *The results of classification achieved at the end of the discriminant analysis*

	Low achievement		High achievement		Total	
Group	*f*	%	*f*	%	*f*	%
Low achievement	116	77.3	14	22.7	150	100.00
High achievement	17	17.3	81	82.7	98	100.00
Total correct classification percentage = 79.4%						

According to Table 5, the types of error made during problem solving correctly classified the low-achievement group at 77.3% and the high-achievement group at 82.7%, while the contribution made by the discriminant function to the classification was calculated to be 79.4%. Given that it is thought that the chance of correct classification was 60.48%, the discriminant function made the correct classification beyond this chance. The third question posed by the study was "Is scaffolding effective in eliminating errors made during the modeling process?" In this context, the changes in error ratios after the 30-student clinical interview study group did the test and received tutor support are presented in Table 7.

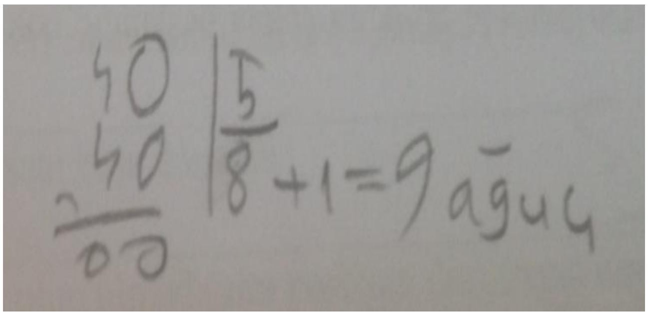
Table 7. *Pre-and post-scaffolding change*

Solutions		Pre-scaffolding	Post-scaffolding: Number of errors corrected	Post-scaffolding: Number of errors not corrected	Scaffolding Success Rate
Missing critical information	N	24	24	0	100.00
	%	8.00	8.00	0	
Incorrect relations	N	35	26	9	74.28
	%	11.67	8.66	3.00	
Incorrect diagrams	N	23	20	3	86.95
	%	7.67	6.66	1.00	
Mental incapacity	N	24	12	12	50.00
	%	8.00	4.00	4.00	
Number consideration model	N	32	21	11	65.62
	%	10.66	7.00	3.66	
Number operator model	N	54	0	54	0.00
	%	18.00	0.00	18.00	
Blank	N	54	-	-	-
	%	18.00	-	-	
Correct	N	54	-	-	-
	%	18.00	-	-	
Total	N	300	103	89	53.64
	%	100.00	34.33	29.66	

According to Table 7, 18.00% of the solutions were left blank and 18.00% of the solutions were correct after the application. When the correct solutions and the solutions left blank were excluded, a total of 192 incorrect solutions were provided with tutor support, 34.33% of which were corrected and 26.66% of which could not be corrected. The next stage of the research includes the example solutions that serve as the main reference point for classifying errors and identifying whether scaffolding was effective.

Errors originating from missing critical information

The example solution for the fourth question, in which the errors originating from missing critical information were most prevalent, and the related clinical interview performed with the student is given in Interview 1.



T: What is being asked of you in the fourth problem?

S: It is asking how many trees need to be planted.

T: Can you solve the problem again vocally?

S: I divided 40 by 5 because the trees will be planted at intervals of 5 meters. Then, I added 1 because one tree is needed at the end.

T: Do you think "9" is correct? Could you read the question again?

S: I think so.

T: Can you read it carefully?

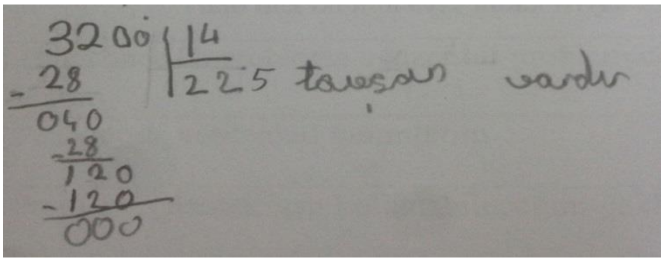
S: They will be planted on both sides. I should have multiplied 9 and 2. (The student emphasized the phrase "both sides.")

Interview 1: *Example of an error observed in the fourth problem demonstrating errors originating from missing critical information*

According to the Interview 1, the student forgot "*the trees will be planted on both sides of the road*" due to fast reading and made the error in the "understanding" phase. It was determined that the students noticed their errors easily and could correct them when the interpretation and verification modeling steps were activated during the clinical interviews. Scaffolding is most successful at correcting errors originating from missing critical information. According to Table 7, it can be seen that this type of error had been made by the pilot group 24 times in the beginning and all of them were corrected through leading questions.

Errors originating from incorrect relations

The example solution for the third question, in which the errors originating from incorrect relations were most prevalent, and the related clinical interview performed with the student is given in Interview 2.



T: What is being asked of you in the third problem?

S: How many rabbits there were in the first year.

T: Can you solve the problem again vocally?

S: There are 3200 rabbits at the end of the seventh year. It doubles each year. It increases 14 times in seven years. I divided 3200 by 14 to find 225.

T: If there were 225 rabbits in the first year and it doubled each year, would there be 3200 rabbits in the seventh year?

S: Yes (The student found 3200 by multiplying 225 with 14).

T: Let us take it one year at a time to see if we will have the same result.

S: Okay. 450, 900, 1800, 3600.

T: You just exceeded 3200 in the fifth year. I think something is wrong. How about reducing per year?

S: How?

T: How does the number of rabbits increase each year?

S: It doubles. Hmmmm. We increase it by doubling per year (Seventh year, 3200; sixth year, 1600; fifth year, 800……first year, 50. It is 50, then.

T: Is it correct? Could you check it?

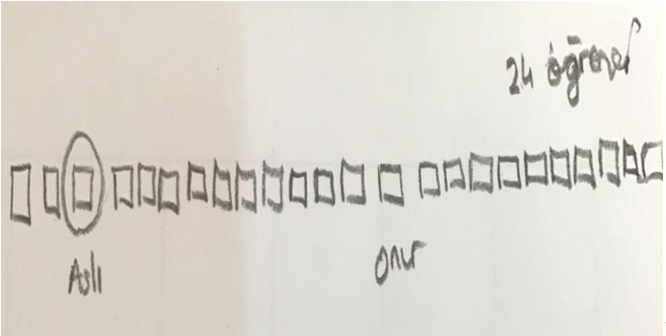
S: 50, 100, 200, 400, 800, 1600, 3200. Yes, it is.

Interview 2: *Example of an error observed in the third problem demonstrating errors originating from incorrect relations*

According to the Interview 2, the student thought "*if it doubles each year, it increases 14 times in seven years*" and could not relate the changes per year to the number of rabbits and so made an error in the structuring step. As can be understood from the example, the main reason for this error is structuring. The students were asked leading questions about the incorrectly related part of the situation model, which ensured that they crosschecked their answer. 26 out of the 35 incorrect solutions made by the pilot group were corrected with tutor support. The success rate of tutor support is 74.28% (26/35) for this type of error.

Errors originating from incorrect diagrams

The example solution for the first question, in which the errors originating from incorrect diagrams were most prevalent, and the related clinical interview performed with the student is given in Interview 3.



T: What is being asked of you in the first problem?

S: To find the total number of students in the line.

T: You drew a diagram to solve it. Can you walk us through it?

S: Aslı is third from first; I placed her in third place. There are 10 people between Onur and Aslı. I drew 10 people between them and placed Onur there because he follows them. Next, I drew 10 people following Onur because there should be 10 people behind him. Then, I counted and found 24.

T: Where is Onur in the line according to the question?

S: In the centre (the student emphasized the phrase "in the centre".

T: What place is Onur in according to your diagram?

S: 14th.

T: If there are 24 people in the line, can Onur, who is in 14th place, be in the centre of the line? How about counting it in the diagram?

S: In 14th place.

T: How many people are there behind him?

S: 1, 2, 3, 4……10 people.

T: Can he be in the centre in your diagram?

S: No.

T: Is there anything wrong in your diagram, then?

S: Yes. I did not draw the people in front of Aslı.

T: T: How many people should there be, then?

S: 27.

T: Is Onur in the centre now?

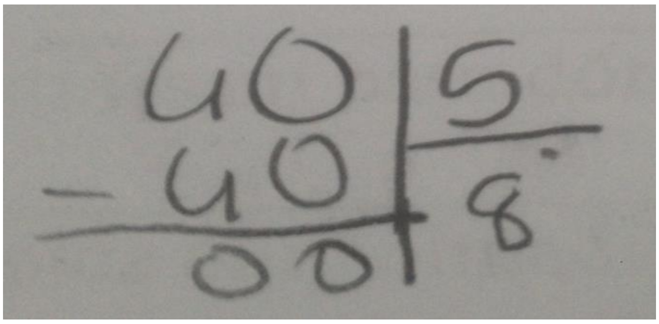
S: Yes. Onur is in 14th place. There are 13 people each in front of and behind him.

***Interview 3:** Example of an error observed in the first problem demonstrating errors originating from incorrect diagrams*

According to Interview 3, the student drew the line in front of Onur correctly, but the line behind him incorrectly. As demonstrated by the example, the error occurred when the student drew the diagram incorrectly after illustrating the situation model incorrectly. Although the error was noticed during external representation, the clinical interviews showed that the errors started off with internal representation. The students were asked questions about the incorrectly drawn part of the diagram, and 20 out of 23 errors in the pilot group were corrected with tutor support. The success rate of tutor support is 86.95% (20/23) for this type of error.

Errors originating from mental incapacity

The example solution for the fourth question, in which the errors originating from mental incapacity were most prevalent, and the related clinical interview performed with the student is given in Interview 4.



T: What is being asked of us in the fourth problem?

S: To find the number of trees.

T: Can you walk me through the operation you performed for the solution?

S: I divided 40 by five to find eight.

T: Why?

S: Because trees will be planted along the 40-metre-long road at intervals of 5 meters. To find how many trees it will take.

T: How many trees would it take if the road was 5 meters long?

S: One.

T: How did you find it?

S: I divided five by five.

T: From which point will the trees begin to be planted?

S: The front end.

T: We planted a tree in the front end. What should we do 5 meters later?

S: Plant one more tree. Then, we have planted two trees.

T: What if the road was 10 meters long?

S: Three (the student used his/her fingers.)

T: 15 meters?

S: Four, one more each time.

T: 40 meters?

S: Nine.

T: Can you read the question again? What does the question ask about the side trees will be planted on?

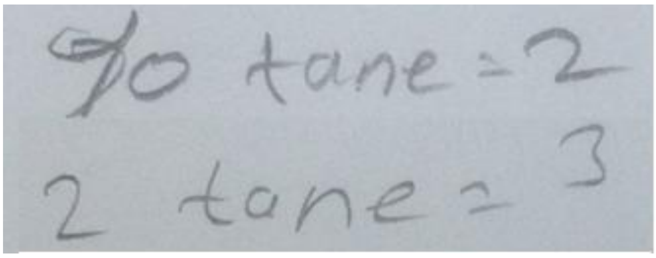
S: On both sides. 18 trees are needed.

Interview 4:*Example of an error observed in the fourth problem demonstrating errors originating from mental incapacity*

According to Interview 4, the student performed one of the operations necessary for the solution but could not continue because s/he did not understand the problem or believed that the solution was finished. These errors originated from "understanding." Tutor support was provided to help to complete the missing parts of the solution, with a success rate of 50% (12/24).

Errors originating from number consideration

The example solution for the sixth question, in which the errors originating from number consideration model were most prevalent, and the related clinical interview performed with the student is given in Interview 5.



T: What is being asked of us in the sixth problem?

S: To find how many tricycles there are?

T: Could you explain the solution?

S: I found it by trial and error. There are 26 tyres. Because there are 10 bicycles, that means 20 tyres. There should be two tricycles so that there can be 26 tyres in total.

T: How many bicycles and tricycles are there in your solution?

S: 12.

T: How many bicycles and tricycles are there in the problem?

S: 10.

T: How many did you use?

S: 12. I tried it wrong.

T: Can you solve it again?

S: If there are 8 bicycles, it is 16 tires; if there are two tricycles, and it is 22 in total, not 26.

T: I suppose the number of tires is not enough, how can we increase it?

S: By increasing the number of tricycles.

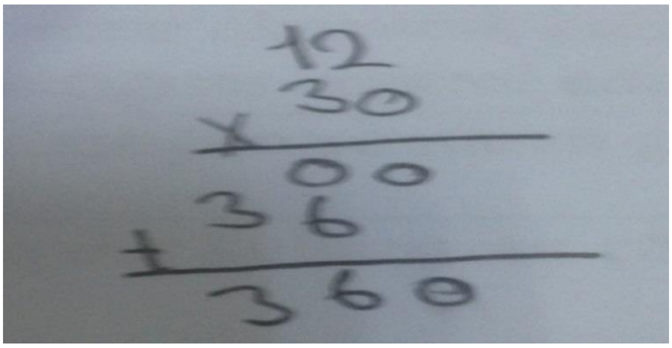
T: Let us continue. If there are three tricycles and nine tires, there will be seven bicycles and 14 tires; it is 23 in total, so this doesn't work. If there are five tricycles and 15 tires, there will be five bicycles and 10 tires; it is 25 in total, so this doesn't work. If there are six tricycles and 18 tires, there will be four bicycles; it is 26 in total now. There should be six tricycles.

Interview 4: *Example of an error observed in the sixth problem demonstrating errors originating from number consideration model*

According to Interview 5, the students only paid attention to the condition of there being 26 tyres, ignoring the condition of there being 10 bicycles while performing the equation. The example and clinical interview demonstrate that students making this type of error could develop suitable strategies for the solution but could not perform an equation that met all of the conditions imposed by the situation model. First, the tutor had the students crosscheck their solutions to make them understand that their answers were wrong. They then had them try different numbers until they found the correct answer so that they could comprehend the conditions of the problem in a holistic way. 21 out of the 32 incorrect solutions in the pilot group were corrected with tutor support. The success rate of tutor support is 65.62% (21/32) for this type of error.

Errors originating from the number operator model

The example solution for the fifth question, in which the errors originating from number operator model were most prevalent, and the related clinical interview performed with the student is given in Interview 6.



T: What is being asked of you in the eighth problem?

S: To find how many rabbits there are.

T: Can you explain the operation you performed?

S: I multiplied 12 by 30 to find 360.

T: Why did you do that?

S: I do not know. It made sense.

T: What does "360" mean here?

S: The number of rabbits.

T: You wrote there are 360 rabbits. But, how many heads are there in the farm?

S: 30.

T: How many heads do 360 rabbits have?

S: 360.

T: There are 30 heads in the farm but you found 360 heads.

S: Yes.

T: Would you like to solve it again?

S: Yes. I need to add.

T: Why?

S: The number will be too high if I multiply.

T: What is it if you add?

S: 42.

T: 42 is higher than 30, too.

S: Yes. I need to subtract, then. 18

T: If there are 18 rabbits, how many feet will there be?

S: I do not know. I do not understand.

Interview 6: *Example of an error observed in the fifth problem demonstrating error originating from the number operator model*

According to Interview 6, the students performed operations which produced results not even remotely close to the solution without understanding the situation model. The tutor support for these situations started with them noticing that the operation they had performed was not reasonable and partial success was achieved. However, this type of error was observed in 54 solutions in the pilot group but none of them could be corrected. In this context, the tutor support was ineffective in correcting this type of error.

Conclusion, discussion and recommendations

Today's mathematical mentality attracts attention to the presence of a dynamic relationship between real-life situations and mathematics, and students are expected to apply their real-life skills to mathematics and vice versa. Blum and Niss (1991) and Lesh and Doerr (2003) define this process as modeling, and MEB (2005) strongly emphasizes the skill of modeling. Despite MEB's strong emphasis on the skill of modeling, the PISA-2015 results reveal that 77.6% of Turkish students are below the third level that shows mathematics can be adapted to real-life. At the end of the study, it was seen that this rate rose to 82% at 4th grade elementary students with solutions left blank. In some studies (Blum & Leiß, 2007; Blum & Borromeo Ferri, 2009; Schapp et al.,2011; Wischgoll et al., 2015; Wijaya et al., 2014) also revealed that students could not evaluate all of the variables together when transforming the real situation into the mathematical model; similar problems were observed in this study.

The clinical interviews revealed that the students would generally give up on the problem during the mathematical result step and ignored two most important steps of the modeling cycle: interpretation and validation. This finding coincides with those from the studies conducted by Maaß (2007), Wijaya and others (2014), Eraslan and Kant (2015). The studies conducted by Teong (2000) and Özsoy and Ataman (2009) also found that using control processes reduced the amount of errors. An examination of scaffolding practices revealed that students managed to notice their errors and correct them when the necessary support was provided for interpretation and validation. This finding demonstrates similarities to the studies performed by Özsoy and Ataman, and Teong. The study determined that the success rate of scaffolding was 34.33% over the test as a whole and 53.64% for eliminating errors. An increase in the success rate from 18% (54) in the pre-scaffolding period to 52.33% in the post-scaffolding period shows that this method can be used to eliminate errors made during modeling. This finding coincides with the study by Wijaya et al*.* (2014).

It was seen that the number operator model was the most frequently made error type (18.22%) by the students during modeling. The clinical interviews showed that this type of error was made because the students made operation choices that produced results not even remotely close to the solution without understanding the real situation. The number operator model is the type of error that most frequently contributes to discriminating between successful and unsuccessful students. In their studies, Pape (2004), Ulu (2016a), Verschaffel et al*.* (1999) and Hong (1993) concluded that unsuccessful students provide number-oriented solutions. The findings of those studies support the findings of this research. Kroll and Miller (1993), Tertemiz (1994) and Prakitipong and Nakamura (2006) found that errors made in word problems are caused by a lack of understanding of the text of the problem rather than basic mathematical skills. The fact that the number operator model, the most frequently made error of the study, occurs due to the understanding step of the process coincides with the findings obtained in studies by Kroll and Miller (1993), Tertemiz (1994) and Prakitipong and Nakamura (2006). The scaffolding provided during the clinical interviews failed to correct this type of error and no errors of this type could be corrected. The scaffolding only helped the students to understand that the results they had found were meaningless. Staub and Reusser (1995) and Kintsch (1988) stated that students should be provided with basic comprehension and inferential comprehension training to eliminate comprehension-based errors.

The fifth most observed type (7.62%) of error in the study is mental incapacity. The clinical interviews revealed that the students making this type of error solved the problem by understanding a little part of it but discontinued solving it because they thought it was the end of the solution or they could not solve it. This finding indicates that this type of error was caused by the understanding step of the modeling process and the subsequent structuring step. Mental incapacity is the type of error that contributes third most to discriminating between successful and unsuccessful students. Scaffolding for this type of error started by making students understand that the solution is incomplete through validation and returning to the real situation. When establishing the real model, local support was applied to the incomplete part, enabling the student to restructure the solution. It was subsequently ensured that the solution was continued by returning to the mathematical model, and the real situation was returned to when necessary. The success rate of scaffolding was 50% for this type of error.

As a result of the study, it was seen that errors originating from number operator and mental incapacity occurred while converting real situation into mental representation of the situation and that these two error types had the rate of 26% in total. In some studies (e.g., Clarkson, 1991; Clements, 1982; Clements & Ellerton, 1996; Marinas & Clements, 1990; Singh et al., 2010; Singhatat, 1991) found that errors in routine problems mostly originated from lack of understanding. The fact that understanding-based errors were the most encountered errors in non-routine problems as a result of the study shows that the research findings correlated to those of these studies.

The second most frequently made student error during modeling is incorrect relations (13.02%). The clinical interviews revealed that this type of error was made during structuring and that it was caused by the fact that the students could not correctly relate the elements of the problem with each other. In studies conducted by Passolunghi and Pazzaglia (2004/2005), it was discovered that these errors were caused by the relation stage rather than the recall step when structuring. The findings obtained through the clinical interviews coincide with the findings of Passolunghi and Pazzaglia's studies. It was found that students who made this type of error performed incorrect internal representations when establishing the real model, which then caused them to perform incorrect external representations when establishing the mathematical model. The error that contributed second most to discriminating between successful and unsuccessful students was found to be incorrect relations, but it was also observed that the majority (74.28%) of these errors could be corrected through scaffolding. The scaffolding was applied to these incorrect relations errors in the mathematical result stage of the modeling cycle and the students were made perform the interpretation/validation stages using leading questions. Using this method, the students were led to understand that their results did not match the real situation. Later, local support was applied to the incorrectly structured part of the solution and the real situation was subsequently achieved by activating the interpretation/validation steps of the modeling cycle following the correct mathematical results.

The least observed type (7.54%) of error in the study is incorrect diagrams. The clinical interviews revealed that students making this type of error used the strategy of drawing shapes and diagrams, but did it incompletely or incorrectly. This finding of the study coincides with those obtained in the studies by Hong (1993), Pantziara and others (2009). The clinical interviews also revealed that those students performed the internal representation during the structuring step of modeling cycle incorrectly, causing an incorrect external representation when establishing the mathematical model. The discriminant analysis showed that this type of error did not contribute to discriminating between successful and unsuccessful students; this indicates that successful and unsuccessful students made this error in similar frequency. The scaffolding provided to students for errors originating from incorrect diagrams started by having the students perform validation so that they could that understand the diagram they had drawn was incorrect, before returning to the real situation. Next, the students were asked to perform the correct internal representation using leading questions and to transfer it with drawings. The scaffolding for this type of error was the second most effective (86.95%) after missing critical information.

As a result of the study, it was seen that errors originating from incorrect relations and incorrect diagrams occurred while converting real model into mathematical model and that these two error types had the rate of 20% in total. In some studies (e.g., Clarkson, 1991; Clements, 1982; Clements & Ellerton, 1996; Marinas and Clements, 1990; Singh et al., 2010; Singhatat, 1991) found that the second most frequent errors of elementary students in routine problems was transformation-based errors. The fact that transformation-based errors were the second most encountered errors in non-routine problems as a result of the study shows that the research findings correlated to those of these studies.

The third most frequent error made by the students during modeling is the number consideration model (13.02%). The clinical interviews showed that the students chose the guess and check strategy to solve the problem, but did not consider all of the conditions in the problem text during the stage of working mathematically. The number consideration model error contributes fourth most to discriminating between successful and unsuccessful students. The scaffolding for students who made this type of error started with going back to the real situation by working backwards from the mathematical result step, therefore helping the students to understand the conditions that needed to be met in the problem text. Next, they were asked to continue the trial and error strategy until they found the correct answer. Scaffolding was effective in correcting this type of error at 65.62%. This finding shows similarities to those obtained in the study by Wischgoll and others (2015). Wischgoll and others concluded that scaffolding was successful for students trying to solve the problem using a trial and error strategy but gave the incorrect answer.

The fourth most observed error types (8.06%) in the solutions are those originating from missing critical information. The clinical interviews revealed that students making this type of error managed to choose and work with the right strategy, but they ignored one of the operations during the solution process because they read a part of the problem incorrectly or incompletely due to lack of attention. The results of the discriminant analysis performed to investigate the effect of errors made during modeling on discriminating between successful and unsuccessful students showed that this type of error has a negative relationship with the discriminant function. This indicates that errors originating from missing critical information are in fact observed among students with a high level of achievement. This finding of the study coincides with those obtained in the study by Wijaya et al. (2014). The clinical interviews revealed that students making this type of error had high confidence and were sure of their results, but overconfidence caused simple errors. These students were provided with scaffolding by going back to the real situation model of the modeling cycle, therefore helping them to understand the points which they did not notice in the problem text. Next, correction of their errors was ensured by guiding them towards the mathematical model. The errors were easily corrected because this type of error was mostly made by the successful students; the success rate of scaffolding is 100% for this type of error.

By the end of the study, it was observed that using the modeling cycle during the process of solving word problems may succeed if a little support is given in the beginning. It is thought that students will make a habit of using all the stages of this process and become more successful solvers when problem-solving activities are performed on the basis of the modeling cycle. In this context, it is recommended that the modeling cycle should be used and scaffolding performed when necessary during in-class modeling activities. This recommendation originates from the fact that students did not interpret and validate the problems in the study.

Providing students with comprehension training can be recommended for eliminating errors originating from the number operator model, which scaffolding was not successful in correcting. It has been observed that the students gave incorrect answers or discontinued their solutions because they could not relate what was given in the problems correctly. Such students can be provided with multi-representational (verbal, visual, symbolic) training during in-class activities. Given that errors originating from incorrect diagrams did not contribute to discriminating between successful and unsuccessful students, this indicates that successful students still had trouble with drawing diagrams. A stronger emphasis on the strategy of drawing shapes and diagrams during in-class activities is therefore recommended. Since it was observed that some students could find the correct answer by using the trial and error strategy and yet some students, especially those with lower modeling achievement, found the incorrect answer despite choosing the right strategy, the provision of training to enhance this type of strategy during in-class activities is recommended. The fact that errors originating from missing critical information were most frequently made by students with higher modeling achievement reveals that the reason for this error is overconfidence. These students should be reminded that they may make simple errors unless they are observant.

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