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Investigation on How Pre-service Elementary Mathematics Teachers Write and Use Mathematical Definitions *

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Abstract

The aim of the study was to determine whether pre-service elementary mathematics teachers could write and use mathematical definitions. For this reason, the question sheet containing two questions, was given to 76 pre-service teachers in a public university who was taking general mathematics course. Qualitative research method was used during data analysing. According to the answers given by them, five themes for the first and six themes for the second question were classified. In order to reach detailed results, semi- structured interviews were conducted with one pre-service teacher for each theme. Findings showed that they had difficulties in writing a formal definition assumed to be known and in using a given definition assumed to be not known of a mathematical concept.

Keywords: Writing and using mathematical definitions, Pre-service teachers.

Introduction

Mathematics course consists of three elements as concepts, algorithms which are needed to solve problems and practice of those algorithms (Selden & Selden, 2003). In order to use those concepts in related areas and places, it is needed to be learned the definitions well. Since the definitions are quite important for being constructed the mathematical concepts and distinguishing a concept from others (Çakıroğlu, 2013).

Tall and Vinner (1981) define the meaning of definition as form of words that explain a concept. According to Borasi (1982), there are some important points for definitions which are that the words in the definitions must be known before, all properties related to concept must be presented and only necessary terms and properties must be given, there must be no conflict between those properties and the concept to be defined must not contain itself as a word.

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There are different views in the literature on how definitions that are so important for mathematics are learned. For example, the importance of personal experiences (Shield & Swinson, 1997) and given examples (Van Dormolen & Zaslavsky, 2003) are mentioned while definitions are being learned. Shield and Swinson, (1997) state that after experiences on a topic related to mathematics, writing definitions strengthens students' understandings. Van Dormolen and Zaslavsky (2003) emphasize that the best starting to learn a new topic is some examples which are selected, and informal definitions about that topic. Adams (2003) indicate that student are asked to test if given examples are suitable to related definition or not, is the one way of getting them to develop definitions and it can be accepted that students use the informal definitions while transmitting to formal definitions.

There are various researches on writing, understanding and using of mathematical definitions. Both in high school level (Rasslan & Tall, 2002) and in undergraduate level (Edwards & Ward, 2004; Zazkis & Leikin, 2008) it is indicated that students have difficulties in definitions of mathematical concepts and usage of them. When researches related to pre-service teachers are analyzed, it is seen that they are incapable of explaining, exemplifying the definitions on geometrical concepts and of using mathematical language (Alkış-Küçükaydın & Gökbulut, 2013; Aslan-Tutak, F., & Adams, 2015; Bozkurt & Koç, 2012). In addition to that, they have difficulties in distinguishing the differences between the formal definition and properties of a concept and in determining the relations between concepts (Kaplan & Hızarcı, 2005; Kar, Çiltaş & Işık, 2011). Another study conducted by Zazkis and Leikin (2008), shows that examples on definition of a square that was written by pre-service teachers, are pedagogical rather than mathematical. To sum up, it is detected that as result of those studies, pre-service teachers have some problems on the knowledge about mathematical definition and usage of them.

Vinner (1991) indicates that, people who study mathematics must know mathematical definitions and how those definitions are used. Since definitions of mathematical concepts, its structure and making definition process are the elements of a mathematics teacher's content knowledge (Zazkis & Leikin, 2008). For that reason, it is expected that teachers and pre-service teachers must know and use definitions of concepts efficiently and realize the relations between different definitions.

In this study, the abilities of pre-service elementary mathematics teachers to write and use mathematical definitions were analyzed. Unlike other studies, in addition to investigate the situation of them, writing a definition of mathematical concept that is assumed to be known before, their situations about using and testing a given definition that they did not know before, were also analyzed.

Research Question

What are the difficulties on that the elementary pre-service mathematics teachers' experiences while they are;

- writing formally the definition (definition of bounded function) given before in the general mathematics course given them lecture,
- testing and using a given mathematical definition (definition of algebraic number) which is thought that they have not known it before?

Method

Qualitative research method was used in this study. In addition to written data collection tools, interviews were also conducted. For qualitative research method, using different data collection tools is one way of providing validity and reliability (Yıldırım & Şimşek, 2005).

Research Group
The study was conducted with totally 76 pre-service teachers, 59 of them were female and 17 of them were male who are studying in a public university. It was paid attention that they were enrolled general mathematics course.

Data Collection Process and Tools
Research consisted of two steps because there were two different data collection tools. In the first step, written data was collected through a question sheet. After those written data was grouped into themes, semi structured interviews were conducted with one pre-service teacher for each theme in the second step.

During the first step, two questions were asked and the answers were wanted in written form. One of the questions was '*write formal definition of bounded function and give an example'*. Bounded functions as a subject is included in the context of the general mathematics course. Formal definition of bounded function is stated and explained by examples in that course. The aim of this question is to determine whether pre-service teachers can write the formal definition of the mathematical concept that is assumed to be known.

The second question is related to algebraic number definition. The definition of algebraic number which is 'roots of polynomial with rational coefficients are called algebraic number.' was given to them and asked them to *'Determine if* $^1/_3$, $\pi - 1$, $\sqrt{1+\sqrt{2}}$, $\sqrt[3]{5}$ *and* $3.\bar{4}$*are algebraic or not.'* The aim of this question was to determine how pre-service teachers use and test the formal definition of a mathematical concept that has not been given before at the general mathematics course and hence it is assumed that they have seen the definition for the first time.

In the second step, in order to elaborate on this process, semi structured interviews were conducted on their written definitions and exemplifications with one pre-service teacher for each theme, specified for both questions. Interview questions are on their writings and testing the definitions of given concepts. Interviews were conducted in a place where one pre-service teacher and the researcher were alone. It was recorded after taking their permissions.

Findings

Findings were presented under two subtitles in terms of writing the formal definition of bounded function and testing the given definition of algebraic number.

The mathematical definition which was assumed to be known before by pre-service teachers: 'Write the definition of bounded function and give an example.'

Before this practice at general mathematics course, the definition of a bounded function was given in the following way; first, definitions of functions which are bounded from above and/or below, were stated after related examples. Then a bounded function definition was given in a sentence as *'a function is bounded if it is bounded from below and above'.* As an example, it was mentioned that *'if a function has maximum and minimum values, then it is naturally bounded between those values'*. It was also emphasized that if a function had no maximum and/or minimum values, it could be a bounded function. Finally, the definition was given as *'Let A be the domain set of the function f.* $\forall x \in A$*, if* $\exists K \in R$ *such that* $If(x)I \leq K$*, then f is called bounded.'*

Written answers were analyzed and grouped under five themes. Only one pre-service teacher could write the formal definition correctly and give a true example, while five of them could not. That's why grouping the answers into themes were done with 70 pre-service teachers' writings. Frequency and percentage table related to these themes was given in Table 1.

Table 1. *Frequency and percentage distribution according to themes*

Theme		Pre-service Teachers *f*	Pre-service Teachers *percentage*
Theme 1	bounded domain	14	20
Theme 2	bounded range	14	20
Theme 3	function with maximum and minimum	21	30
Theme 4	writing the definition partially	4	5.7
Theme 5	definitions that are ungrouped	17	24.3
	Total:	70	100

Considering Table 1, it was interesting to see 30% of answers given as 'to be a bounded function it shall have maximum and minimum values.' On the other hand, writing the formal definition even partially took only 5% of them. In addition to that, approximately 25% of answers could not be categorized into any theme.

Interviews were conducted with one volunteer pre-service teacher for each theme in order to examine their answers in details and findings were presented for all themes separately below.

Bounded Domain

In this theme, a written definition and its example were given at a pre-service teacher's answer sheet as indicated at Figure 1.

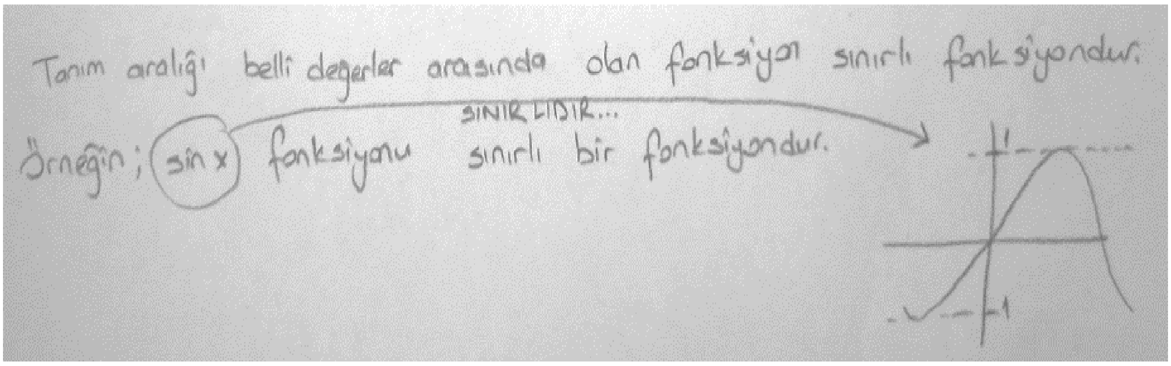

Figure 1. *The written definition of pre-service teacher*

Writings in Figure 1 could be translated as, *'If the domain of a function is between certain values, it is a bounded function'.* For example *sinx* was a bounded function because function values were in (-1,1). Also a part of the graph sin function was drawn.

We emphasized here that she wrote a wrong definition but gave a true example of a bounded function. So it was understood that she was not aware of incompatibility of the definition and example that she gave. During the interview with her, after her awareness of this incompatibility with the help of researchers' explanation, she said *'I stated definition wrongly there, but I want to mean the values that function takes, must be in a certain intervals.'* When another example was asked for a bounded function, she said *'We learnt this example at the course and I do not know another example. ''x takes values in a certain intervals, for instance -2< x <8'* was another example given by them. In this theme, definitions, written by them, were roughly *'functions whose domains are finite intervals, are bounded.'* But it was obvious that there were unbounded functions whose domains were bounded. For instance $f_{(x)} = \dfrac{1}{x-1}$ was not bounded in (0.1).

Bounded range

14 pre-service teachers wrote some kinds of informal definitions. Although two of them gave true definition of a bounded function as a meaning, they wrote examples of unbounded functions like *tanx* or $\sqrt{x}$.

A pre-service teacher used 'bounded range' expression in his writings. When the meaning of bounded range was asked to the pre-service teacher in the interview, he said it meant *'the values it takes, must be in a certain range'.* From this explanation, it could be thought that he knew the definition, but the example which was given to support his definition, was not proper. Since in his answer sheet the sentence *'for example* $\sqrt{x}$ *is a bounded in/on [0, ∞)'* (it was given in figure 2) conflicts with his definition. In order to clarify this confliction in his mind, some questions were prepared to ask in the interview but he was not willing to answer them.

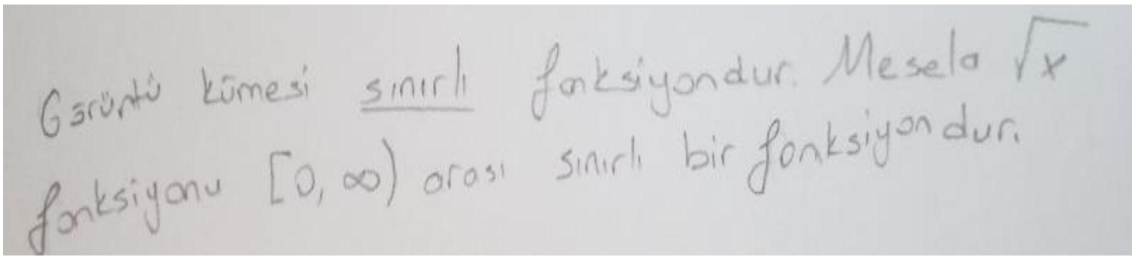

Figure 2.*The written definition of pre-service teacher*

The definition, written by them, in this theme were roughly 'functions are bounded if their ranges are bounded.'

Function With Maximums and Minimums

The definitions, written by pre-service teachers in this theme were roughly in the form 'function defined in the interval I is bounded between its maximum and minimum values.' Five pre-service teachers needed this interval I to be a closed finite [a,b] interval while the others did not need such a condition on domain of the function. Two of them from those five took the maximum and minimum values of *f* at the end points of the interval a and b, respectively, while other three showed maximum and minimum values of *f* at interior points of the interval [a,b] in their drawings. The interesting thing here was that where maximum and minimum values were taken was not important, but domain of the function was restricted as a finite interval.

The misconception of pre-service teachers in this theme was that a function to be bounded needed to have maximum and minimum values. That's why in the interview, it was asked a pre-service teacher representing this theme to answer the question *'if the function did not have maximum and minimum points, isn't it bounded?'* The answer is *'No, that time it cannot be a bounded function.'*

Writing the Definition Partially

There were four pre-service teachers in this theme who tried to write the formal definition but they could not write it accurately. It was restated the formal definition of a bounded function to remember: *'Let A be the domain set of the function f. $\forall x \in A$, if $\exists K \in R$ such that If(x)I ≤ K.'* Only one pre-service teacher could write '$\forall x \in I$' symbolic expression which was needed to write a part of the definition, but none of them could state the symbolic expression 'if $\exists K \in R$' in their definitions.

The answer sheet of a pre-service teacher in this theme was given in Figure 3. In the paper *'Let If(x)I ≤ K and I =[a,b]. If K $\in$I, then f is bounded on I.'* and this definition was illustrated by an example consisting of an inequality and a graph. Being the expression 'K $\in$I' in her answer sheet showed that she was not aware of what she defined. That's why in the interview, questions like '*In which set K should be?, Is it a member of a set or just a real number?'* were asked. She did not answer it. Later, it was asked her to write the definition again. Thereupon, she said that *'I get used to solve question, writing definition is hard for me, I memorize* and *write it.' and 'I memorized it before the exam. But I have forgotten it, so I cannot write it again.'*

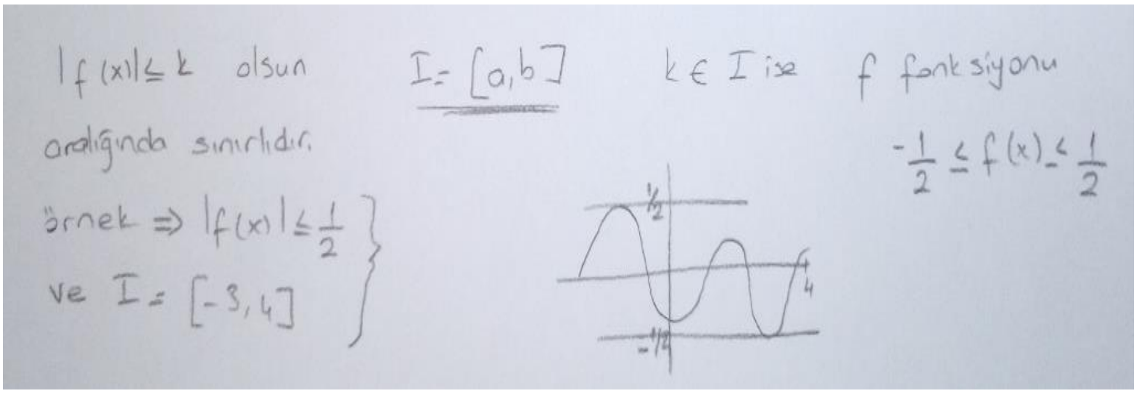

Figure 3.*The written definition of pre-service teacher*

Definitions That Are Ungrouped

This group consisted of answers which could not be classified in the question sheets. Since the answers included statements which were unclear, nonsense, unnecessary and had complicated mathematical concepts. One of these answers was *'No matter how many elements exist in the domain of function, if the number of elements in the image set is constant, then function is bounded'* in this theme. In addition to that, there were pre-service teachers who could not make definition, tried to explain a bounded function by drawing. Apart from that, some of them thought bounded function as *'restricted function'* or as *'piecewise function'*. For instance, a pre-service teacher defined the bounded function as *'the function, obtained by applying some restrictions or conditions to an equation, is called a bounded function'* and exemplified it as *f(x)=2-*$\sqrt{-x^2+5x-4}$,*2 < x ≤ 3'.* Another pre-service teacher defined it as *''bounded function is to define a function between specified bounds'* and tried to illustrate his definition by his drawings, given in Figure 4.

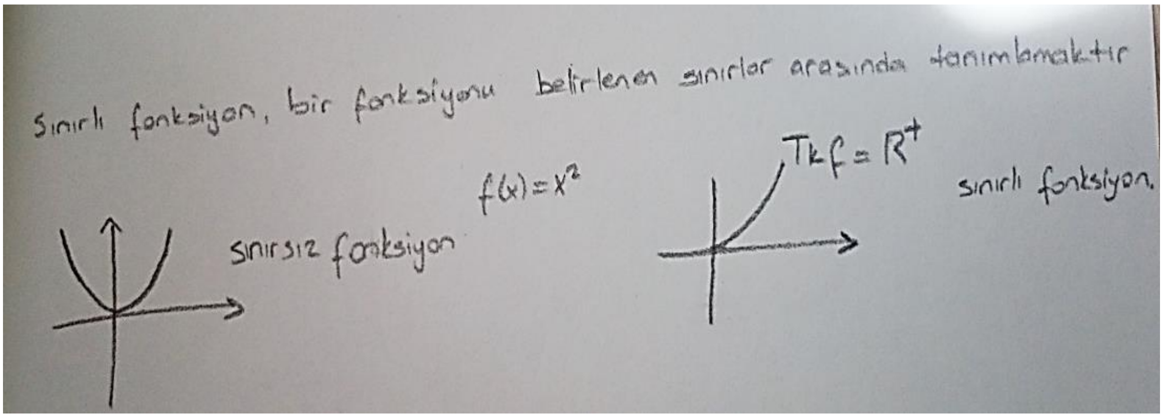

Figure 4.*The written definition of pre-service teacher*

It was assumed that the mathematical definition hadn't been known before by pre-service teachers: 'Roots of polynomial with rational coefficients are called algebraic number. Determine if $1/3$, $\pi - 1$, $\sqrt{1 + \sqrt{2}}$, $\sqrt[3]{5}$ *and* $3.\overline{4}$ *are algebraic or not?'*

Any information about algebraic number hadn't been given in the general mathematics course. That's why it was assumed that they didn't know the definition of algebraic number. This assumption was questioned and verified during semi structured interviews.

The definition of algebraic number was written in question sheet. It was asked them to test definition for the given five numbers whether they were algebraic or not. Written answers were grouped under six themes.

Two of them from 76 pre-service teachers didn't give any answers for this question. That's why grouping of the given answers was done with 74 answers. Related frequency and percentage distribution were given in Table 2.

Table 2. *Frequency and percentage distribution of the themes*

	Theme	**Pre-service teachers**	
		f	***percentage***
Theme 1	using the words in definition without any explanation	18	24.3
Theme 2	just giving polynomial satisfying the definition	7	9.5
Theme 3	if rational/irrational then algebraic/not algebraic	17	22.9
Theme 4	given numbers calculated with algebraic operations	13	17.5
Theme 5	mixed theme	7	9.5
Theme 6	statements without mathematical base	12	16.3
	Total	74	100

When the values were analyzed in the table 2, it was seen that the percentages of theme 1 and 3 were approximately same (24%). Besides this, the percentages of theme 4 and 6 were about 17%. Interviews were conducted with pre-service teachers for each theme in order to examine their answers in details. Findings were presented for all themes as followings.

Using the Words in Definition without Any Explanation

The pre-service teachers in this theme answered the question by using the words in the given definition and didn't support their statements with any other expressions. Some of them labeled the numbers only as *'algebraic'* or *'not algebraic'*. A few of them selected some numbers and wrote that '*they are algebraic because they can be roots of a polynomial with rational coefficients'.* But they didn't write any polynomial whose roots are the numbers they selected and claimed as algebraic.

Answer of pre-service teacher interviewed for this theme was presented below. He just stated $1/3$, $\pi - 1$ *and* $3.\overline{4}$ were algebraic.

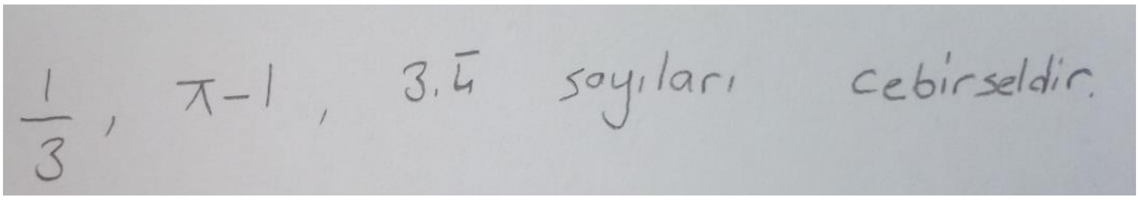

Figure 5.*Written Answer of Pre-service Teacher*

As it was seen in the figure there was no explanation related to the given answer. When it was asked how she reached the result, she said that *'I did not understand while I was writing it, I concocted. '*In addition to that, pre-service teacher's expression which was *'If I wrote π as 3.14, I could say it is a polynomial. I cannot write now since it is π.' are so remarkable.*

Just Giving Polynomial Satisfying the Definition

While some of them in this theme could give correct polynomial examples for a few algebraic numbers, the others gave wrong examples. There was only one pre-service teacher who could give true polynomial example for three algebraic numbers which were *if* $1/3$, $\sqrt[3]{5}$ *and* $3.\overline{4}$. Answer of pre-service teacher interviewed for this theme, was presented in Figure 6.

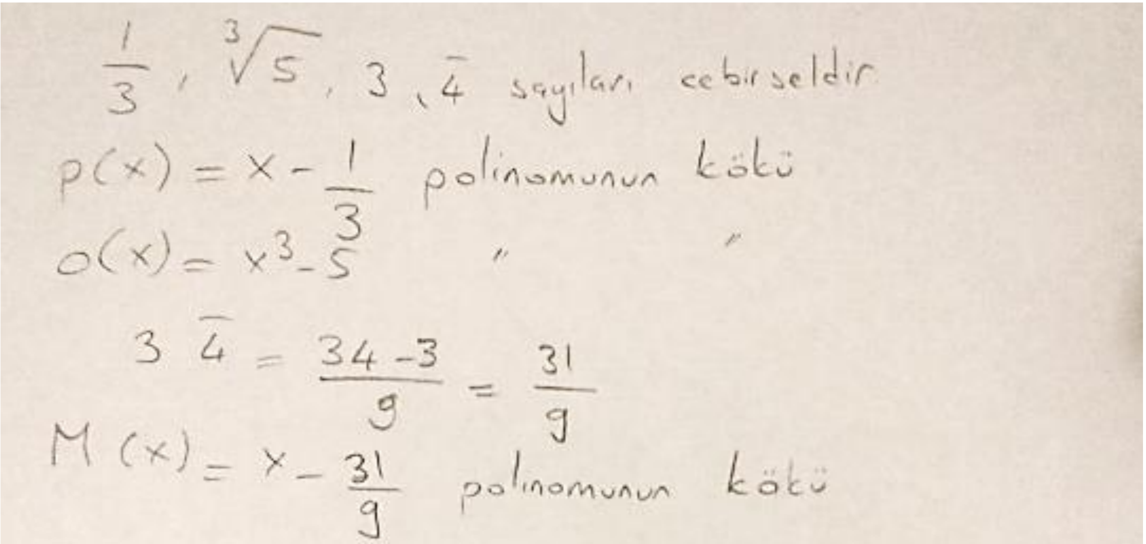

Figure 6.*Written Answer of Pre-service teacher*

As it was seen in the figure, she didn't deal with the numbers $\sqrt{1+\sqrt{2}}$ and $\pi - 1$. In the interview it was asked the reason of her ignorance and wanted her to test those numbers again. Despite of the guidance of researchers, she couldn't determine whether the numbers were algebraic or not.

If Rational/Irrational Then Algebraic/Not Algebraic

In this theme, pre-service teachers believed mistakenly that rational or irrational numbers must be an algebraic or no algebraic number, respectively. It was very interesting to see this wrong idea was used by approximately $1/_4$ of them. In the interview, the reason about why such a classification was needed and the given definition of algebraic number was not used, was asked to the pre-service teacher. He said that *'the irrational numbers cannot be algebraic because it is written in the definition that the coefficients must be rational in the definition given'.* In figure 7, he classified *if* $1/_3$, $\sqrt[3]{5}$ *and* $3.\overline{4}$ as algebraic because of being rational and $\sqrt{1+\sqrt{2}}$ and $\pi - 1$ as no algebraic because of being irrational.

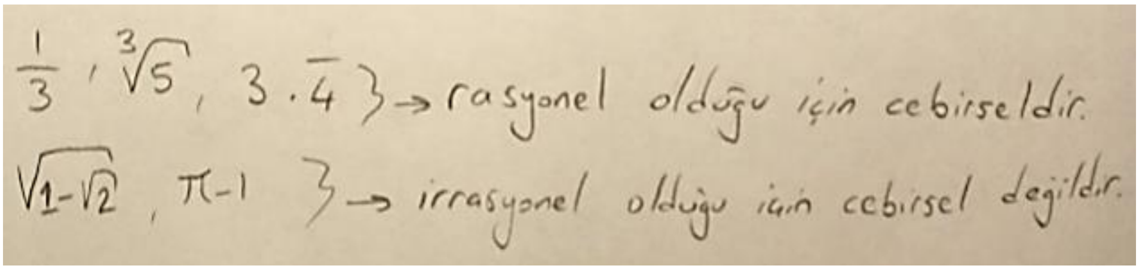
$\frac{1}{3}$, $\sqrt[3]{5}$, $3.\overline{4}$ } → rasyonel olduğu için cebirseldir.
$\sqrt{1-\sqrt{2}}$, $\pi-1$ } → irrasyonel olduğu için cebirsel değildir.

Figure 7.*Written Answer of Pre-service teacher*

By the way, as it was seen in figure 7, he labeled the number $\sqrt[3]{5}$ as rational. By means of this case, it was understood that they had difficulties in classifying rational and irrational numbers.

Given Numbers Calculated With Algebraic Operations

In their answer sheets, it was seen that instead of using given definition, they explained their answers on the base of the sentence *'the numbers or functions are obtained by algebraic operations'.* They thought that the given numbers could be obtained through algebraic operations and those numbers, obtained by this way, were algebraic numbers. A written answer of a pre-service teacher in this theme was given in Figure 8. He redefined the algebraic number as *'numbers, obtained through operations like summation, subtraction, multiplication, division and taking root are called algebraic numbers.'*

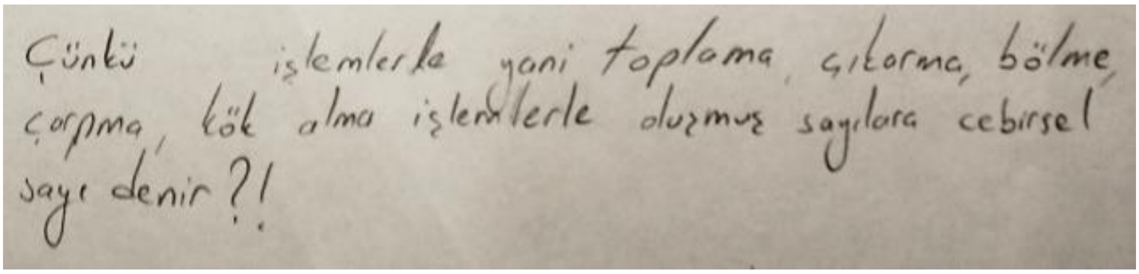
Çünkü işlemlerle yani toplama, çıkarma, bölme, çarpma, kök alma işlemlerle oluşmuş sayılara cebirsel sayı denir ?!

Figure 8. *Written answer of a pre-service teacher*

Mixed Theme

The expressions which were grouped in this theme, could be considered as a combination of other themes. In the interview, it was asked to understand what she thought when she made such a combination and it is answered as *'it must be a rational number to be an algebraic expression. In order to be an algebraic number, it must be obtained with algebraic operations like addition, subtraction, multiplication, division'* This answer could be seen as an example of a combination of theme 3 and theme 4.

Statements without Mathematical Base

The pre-service teachers' expressions in this theme were not related to the concepts in given definition and had not got any mathematical bases. For instance one pre-service teacher understood the word of *'root'* in the given definition, as '*square root of a number*' mistakenly. Pre-service teachers, who thought like this, labeled $\sqrt{1+\sqrt{2}}$and $\sqrt[3]{5}$ as algebraic numbers.

Originating the fact *'the power of a polynomial function must be a natural number'*, one of the pre-service teacher got a wrong inference which was *'exponent of an exponential numbers must be a natural number'.* By this wrong inference, she stated the numbers $\sqrt{1+\sqrt{2}}$and $\sqrt[3]{5}$ as $(1+\sqrt{2})^{1/2}$ and $5^{1/3}$ respectively, and mentioned that those numbers could not be algebraic number since their powers are not natural numbers. One more example for the expressions, written by a pre-service teacher without mathematical base, was *'repeating decimals are not algebraic functions'*.

Discussion and Suggestion

According to the findings, it was seen that pre-service teachers had difficulties in writing definition of bounded function that is assumed to be known before. Those were striking results that only one among them could write formal definition of a bounded function accurately and only four of them could write partially. This finding on their difficulties in writing definitions showed similarity with other researches in literature (Edwards & Ward, 2004; Bozkurt & Koç, 2012). On the other hand, they couldn't state the formal definition although they could give examples of a bounded function. This situation could be interpreted as they were aware of the meaning of definition but they could not write it formally. They tried to express definition informally and supported their definitions with some examples which had been given before at general mathematics courses. Usage of those same examples was given rise to think that they tended to memorize what they write at the course without thinking on it.

Furthermore, they also had difficulties when they were trying to use and to test a given definition. In this study, definition of the algebraic number was given and asked to test five numbers if they are algebraic or not. Approximately only 10% of them could test few numbers.

All pre-service teachers who were interviewed with, stated that writing a definition of a bounded function was easier than using a given definition of algebraic number. They justified the reason of their familiarity to the bounded function definition, from high school to university. From this, it may be inferenced that when they felt more confident in writing a definition that they were familiar with, than testing and using a given definition that they were not familiar with. Even so stated and explained, it should be emphasized again that only one of them could write the formal definition of bounded function.

Moreover, there were some pre-service teachers who wrote their own definition by ignoring the given definition. This situation was similar with the result of Holguin and Selden (2014)'s research. In related study, 23 graduate students ignored the given definition and tended to use their own concept images and old knowledge.

We recalled the definition of algebraic number as *roots of polynomial with rational coefficients are called algebraic number.* For instance 1/3 is an algebraic number since it is the root of the polynomials 3x-1, $x-{}^{1}/_{3}$, $9x^2-1$,... As it was seen that there were many polynomials whose one of the root was ${}^{1}/_{3}$, so ${}^{1}/_{3}$ is an algebraic number. When pre-service teachers were checking the given number *'a'* assumed as an algebraic number, they generally used x-a polynomial form which is the simplest one.

In addition to that, they had some misconceptions and misunderstandings about mathematical concepts apart from the subject of this research. Unfortunately, it was disappointed to see that irrational numbers as rational, at their answer sheets. During the interview, one stated that he still couldn't know whether π was rational or not. The reason of this situation could be interpreted as that they were not sure about their own content knowledge related to these basic concepts. Ubuz and Gökbulut (2015) also mentioned that pre-service teachers had difficulties in defining mathematical concepts, because of their lacking of content knowledge. That's why content knowledge is an essential point for both writing and using a definition. The other point was that when expressions of some pre-service teachers related to algebraic number were analyzed, it was seen that their inferences on definitions were far from their original meanings. For instance, the statement "The roots of a polynomial expression" in the definition of algebraic number was thought as a "square root "or "cube root" of a number.

To sum up, pre-service teachers had difficulties both in writing and in using the definition of a concept that was given. The most remarkable point was even if the definition of a concept was given, they were incapable of testing and using it. However, Zaslavsky and Shir (2005) emphasized that the definitions were the on base of mathematics and mathematics education, it was worrying to see that they had such difficulties. The kind of our education system is about solving many questions rather than learning definitions. We give importance to solve exercises related to a definition instead of using that definition to make the concept clear. For this reason, writing and using the mathematical definitions become more difficult than solving related problem. To overcome with those difficulties, mathematical classroom activities related to the definitions may be included in all levels of our education system. Enough time and attention should be given for definitions during mathematics classes from elementary school to college years. By this way, students may internalize the meaning and usage of definitions. Since the definitions are the fundamentals of mathematics, discussions on definitions can be conducted during the courses to enhance conceptual understanding and use of mathematical language. Abilities like transforming verbal definitions into other forms (symbolic, algebraic, graphical, etc..) should be gained during mathematics classes. Definitions should be linked and exemplified with daily life situations. So they will be more meaningful and permanent.

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