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Screening for Characteristics of Dyscalculia: Identifying Unconventional Fraction Understandings

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Abstract

Researchers intending to identify the unique characteristics of dyscalculia rely upon the problematic and imprecise proxy of low mathematics achievement. Although detailed case studies of adults with dyscalculia have offered insight into its characteristics, we do not yet know if these characteristics are unique to dyscalculia and could be used to screen younger students for these understandings. To address this, we designed a group-administered written assessment based on the unconventional understandings found in adults with dyscalculia to investigate whether these understandings are atypical. In study 1, we assessed 390 grade 6-8 students to investigate the prevalence of these understandings. In study 2, we assessed 80 grade 6-8 students and recruited three students who demonstrated high levels of unconventional understandings. We collected additional assessment data and determined that all three students met stringent clinical dyscalculia criteria. These studies provide a proof-of-concept for designing dyscalculia screeners based on the characteristics identified in adults with dyscalculia.

Keywords:

Math, Learning Disability, Rational Numbers, Assessment

Introduction

Dyscalculia is a cognitive difference in numerical processing that results in persistent and significant problems learning even the most basic mathematics (Butterworth, 2005; Mussolin et al., 2010). It is estimated that approximately 6-8% of school-aged children have dyscalculia, also referred to as mathematics learning disability[1] (Gross-Tsur et al., 1996; Shalev, 2007). Unfortunately, research on dyscalculia has been hindered because of the lack of a validated and reliable assessment to identify students with dyscalculia (e.g., Geary, 2004; Mazzocco, 2007; Price & Ansari, 2013). Researchers currently identify students with this disability by administering a standardized achievement test and selecting a cutoff threshold, below which students are considered to have dyscalculia[2] . There is great variability in the assessments used and the cutoffs selected (Lewis & Fisher., 2016; Price & Ansari, 2013) suggesting that researchers may not be studying one common phenomenon. Of greater concern is the use of low achievement as a proxy for dyscalculia because of the myriad reasons that students may perform at a "low" level on a test. The current identification approach used by researchers cannot differentiate low achievement due to dyscalculia from low achievement due to social, affective, environmental, or instructional factors. Indeed, the use of low achievement to identify students with dyscalculia has resulted in the over-representation of students of color, non-native english speakers, and students from low SES backgrounds in the dyscalculic group (Hanich et al., 2001, e.g., Compton et al., 2012). The findings of studies relying upon this kind of identification approach may reflect characteristics of low mathematics achievement rather than dyscalculia per se. This fundamentally limits the validity of these findings and the field's efforts to delineate the unique characteristics of this disability.

Although students with dyscalculia often do have low mathematics achievement, researchers need a more precise way of identifying students with this disability. The Diagnostic Statistical Manual, Fifth Edition (American Psychiatric Association, 2013; DSM-5) requires that environmental, economic, and instructional factors are ruled out before a dyscalculia diagnosis. Furthermore, the DSM-5 recommends a stricter low achievement criterion – the 7th percentile rather than the more commonly used 25th percentile (see Lewis & Fisher, 2016 for a review). Unfortunately, research on dyscalculia has not moved to adopt these more stringent criteria. This may be partially due to the fact that to allow for statistical comparisons, researchers must ensure that a sufficient number of students meet the study's dyscalculia criteria (e.g., Geary et al., 2000). This may also be due to the fact that differentiating cognitive and non-cognitive causes of low achievement is time consuming, methodologically challenging, and often requires longitudinal data collection (e.g., Mazzocco & Myers, 2003).

To address the need for a dyscalculia screener that does not rely upon low achievement, Butterworth (2003) developed a Dyscalculia Screener. This screener measures the student's speed and accuracy on simple arithmetic and rapid quantity comparisons thought to be associated with number sense (Dehaene, 2011). Unfortunately, researchers have found that this assessment misidentifies students (both false positives and false negatives) based on longitudinal data (Gifford & Rockliffe, 2012; Messenger et al., 2007) and therefore it has not been used in research on dyscalculia.

Because the characteristics of dyscalculia are not yet understood, it remains unclear what measures a dyscalculia assessment should contain (Price & Ansari, 2013). As researchers attempt to identify and define the core characteristics of this disability (Butterworth, 2005), they are doing so with the imprecise criterion of low mathematics achievement. Reliance upon the problematic proxy of low achievement leads to "findings that are difficult to interpret, replicate, and generalize" (Lyon, 1995, p. 7). We argue that accurate identification of students with dyscalculia is the central challenge in this field.

To make progress in understanding the unique characteristics of dyscalculia and improve identification methods, researchers must take a radically different approach. Rather than starting with large samples of students identified with the imprecise proxy of low achievement, it may be more advantageous to start with small samples of extreme cases, as has been productive in defining other disabilities. By "extreme cases," we mean instances in which an individual's physiology or behavior is not aligned with structural or societal expectations and thus it appears to warrant categorization and classification[3]. Detailed study of extreme cases has been essential to identify the defining characteristics of other disability categories, including attention deficit hyperactive disorder (Lange et al., 2010), autism (Wolff, 2004; Verhoeff, 2013) and dyslexia (Duane, 1979). For each of these disabilities, early clinical identification of extreme cases led to defining characteristics of the disability that were used to identify and further refine the definition (e.g., Verhoeff, 2013). For dyscalculia, extreme cases could be adults with a long history of significant and pervasive issues with math (e.g., Mejias et al., 2012), who continue to struggle with arithmetic despite sufficient educational opportunities. Detailed analyses of these kinds of extreme cases can allow researchers to identify characteristic patterns of understandings evident in individuals with dyscalculia. Longitudinal studies have suggested that the difficulties experienced by students with dyscalculia persist over years (e.g., Lewis, 2014; 2017; Mazzocco et al., 2013), suggesting what is learned from adults with dyscalculia could inform investigations with younger students.

In this paper we draw upon characteristics identified in adults with dyscalculia in Lewis's (2014) case study work and design a pencil-and-paper assessment to investigate whether it is possible to identify these understandings in younger students on a group administered written assessment. We designed the written assessment based upon the Lewis (2014) case study for several reasons. First, this is one of the few detailed analyses of extreme cases of dyscalculia – focusing on basic fraction understanding for two adult students. Second, this study used a multidimensional identification approach (see Fletcher et al., 2007) which involved ruling out social and environmental causes for the students' low mathematics achievement, in addition to establishing that these students did not benefit from 1-on-1 tutoring instruction that was effective for younger typically achieving students. Third, common patterns of understandings were identified between the two students with dyscalculia, which were found to provide a productive explanatory frame for unexplained patterns of errors found in longitudinal studies of students with dyscalculia (Lewis, 2016; Mazzocco, et al. 2013). Fourth, these patterns of understandings were evident even in a student with dyscalculia who learned how to compensate effectively (Lewis & Lynn, 2018). Due to the persistence of these patterns of understanding and the commonality across students with dyscalculia, in this study we sought to evaluate how common these patterns were in students in general. The idea being that if these understandings were common in students with dyscalculia, but not typically achieving students, then these understandings could be used to selectively screen students for more extensive assessment and evaluation. The goal is to begin to disrupt the tautological relationship of low achievement and dyscalculia in the field, by identifying behavioral characteristics of the disability itself.

In this section we begin by presenting our sociocultural theoretical framing of dyscalculia, drawing upon Vygotsky's (1929/1993) conception of disability as qualitative human variation. We then describe the patterns of understanding identified in Lewis (2014), and consider these patterns in light of our theoretical framing.

Difference Not Deficit

Vygotsky's theory of disability is focused on understanding qualitative differences and is situated within his general theory of human development. Vygotsky (1981) argued that all human development progresses along two lines: the biological and sociocultural. For typically developing individuals, these two lines of development intersect. The individual's biological development intersects with the sociocultural line of development through social interactions which are mediated by tools (e.g., pencil) and signs (e.g., language). For individuals with disabilities, the sociocultural tools and signs that have developed over the course of human history may be incompatible with the individual's biological development (Vygotsky, 1929/1993). For example, spoken language is not accessible to a Deaf child and therefore does not serve the same mediational role to support the child's development of language as it would for a hearing child. In the case of students with dyscalculia, standard mathematical mediational tools (e.g., numerals, representations) may be incompatible with how these students process numerical information. Vygotsky (1929/1993) argued that this divergence of the sociocultural and biological lines of development does not result in an individual that is less developed, but an individual who has developed differently. This theoretical framing suggests that students with dyscalculia may use and understand standard mediational tools and signs in ways that are qualitatively different from and inconsistent with canonical mathematical usage. Therefore, analytically it is critical to attend to the unconventional ways that students understand and use standard mathematical representations.

Unconventional Understandings Identified in Fractions

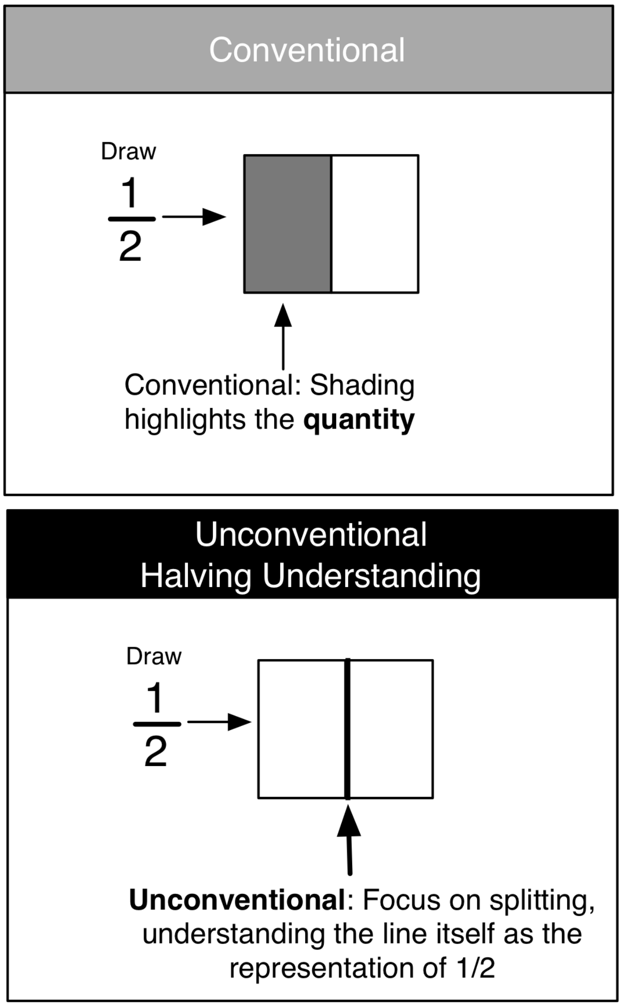
Lewis (2014) identified unconventional fraction understandings in two extreme cases of dyscalculia – two adult students (ages 18 and 19). Both students entered their schooling with considerable privilege, both students were White, upper-middle class, and native English speakers. They attended well-resourced schools and both students had access to additional support and tutoring outside of school. Despite these supports, both students had low mathematics achievement and a long history of difficulties with mathematics which could not be explained by affective or environmental factors. These students also did not benefit from a series of tutoring sessions that were effective for younger typically achieving students (see Lewis, 2014 for details). A detailed analysis of video data from the tutoring sessions on fractions identified a small set of reoccurring and persistent understandings that the students relied upon, which were ultimately detrimental to their learning. These understandings involved using mathematical representations in unconventional ways. Both students had similar unconventional understandings which resulted in a similar pattern of errors. These unconventional understandings involved how students represented and understood the fraction ½ (halving understanding) and how they interpreted fraction representations in terms of the fractional complement (fractional complement understanding).

Unconventional halving understanding

The unconventional halving understanding involved representing the fraction ½ by halving a shape, in which the partition line itself was understood as the representation of ½ rather than 1 of the 2 parts (see Figure 1). For example, when students were asked to draw a picture of ½ they would draw a shape and partition it into two parts. When asked what part of their drawing represented ½, they would point to the partition line itself, often accompanying their explanation with a chopping gesture. Characteristic of this kind of understanding is a focus on the equality or balance between the two parts. For these students ½ was understood as an action, splitting, rather than a fractional quantity (e.g., 1 part out of 2). Although students' experiences splitting, partitioning, and sharing have been shown to be a productive resource upon which students can build (e.g., Empson, 1999; Steffe, 2010; Wilkins & Norton, 2011) the halving understanding was detrimental for both students in that it led to errors and limited the utility of various fraction representations (e.g., area models). Both students understood the fraction ½ as a process, rather than an object (Sfard, 1991), meaning that ½, the most intuitive and best understood fraction (Hunting & Davis, 1991) was not understood as a quantity.

Figure 1

Illustration contrasting the conventional understanding of one-half with the unconventional halving understanding found in students with dyscalculia (Lewis, 2014). Adapted from Difference Not Deficit: Reconceptualizing Mathematical Learning Disabilities, copyright 2014, by the National Council of Teachers of Mathematics. All rights reserved

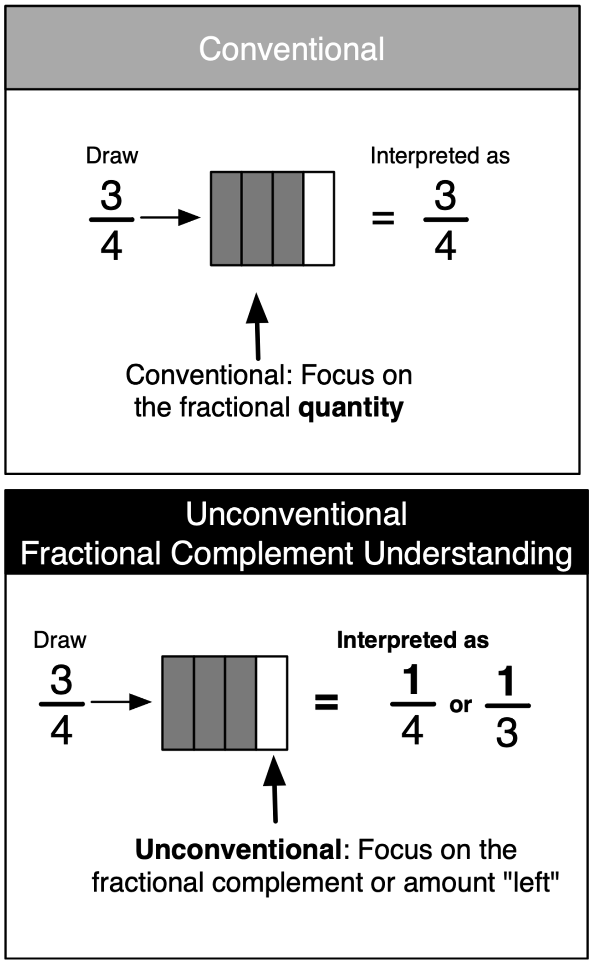


Unconventional fractional complement understanding.

The unconventional fractional complement understanding involved interpreting fraction representations in terms of the fractional complement. For example, interpreting an area model representation of ¾ as ¼ (unshaded/total) or 1/3 (unshaded/shaded), where the unshaded region was understood to be focal (see Figure 2)[4]. Although on the surface this might seem to be an issue of convention – attending to the white rather than shaded parts – for these students it reflected a disconnection between how students constructed and interpreted fractions. For example, when asked to draw the fraction ¾, they would draw a shape, partition it into 4 equal parts, and shade 3 of those parts. However, when asked what their own drawing represented, they would say "one-fourth" explaining that three parts were taken away, and one part was left. This suggested that these students did not have a stable way of representing a fractional quantity and the quantity itself transformed through the act of representing it. Characteristic of this understanding was conceptualizing the shaded fractional quantity as "taken away" or "gone" and referring to the unshaded fractional complement as an amount "left." More telling was that instructional attempts to correct this apparent "mistake" were not successful, even though the students knew that they made these errors, they could not stop themselves from thinking of the shaded as "gone" and the unshaded as "left" (Lewis, 2017).

Figure 2

Illustration contrasting the conventional understanding of area models with the unconventional fractional complement understanding found in students with dyscalculia (Lewis, 2014). Adapted from Difference Not Deficit: Reconceptualizing Mathematical Learning Disabilities, copyright 2014, by the National Council of Teachers of Mathematics. All rights reserved



Issues of access

These unconventional understandings (halving and fractional complement) were evident across a range of different problem types and representations. These understandings appeared when students were working with number lines, concrete fraction representations, and drawn pictures (e.g., area models). These unconventional understandings led to errors and resisted all standard instructional efforts to address them. These understandings were also not evident in typically achieving students who participated in the tutoring sessions. These halving and fractional complement understandings involved an issue of access, where standard mediational tools (e.g., fraction notation, area models) were not serving the purposes they were intended to support. Rather than understanding representations of fractions to show quantity, they understood these representations to show action (e.g., "taking"). Their understandings were, therefore, incommensurate with conventional mathematics use. Perhaps because the students understood fractional quantities as processes rather than objects, they had difficulty using these fractional quantities in other processes (e.g., adding ½ and 1/3 or finding an equivalent fraction for ¾) (Sfard, 1991). Not only did the unconventional understandings persist through the weekly tutoring sessions, but follow up studies suggested that these understandings persisted across multiple years (Lewis, 2017).

The Current Studies

To evaluate the prevalence of these kinds of understandings and the utility of using these characteristics to screen students, we designed a 13-item group administered paper-and-pencil assessment. We refer to this assessment as a "Screener" because we are specifically interested in screening students for halving and fractional complement unconventional understandings. The screener questions were based on questions from Lewis (2014) in which students demonstrated these unconventional understandings. Students were asked to draw, interpret, compare and operate with a variety of fractional quantities. For a complete list of questions with scoring guide see Appendix A. The screener questions were deliberately designed to elicit evidence of halving or fractional complement understandings, therefore, we did not specify the manner in which students should interpret fraction representations. Students were given one unconventional understanding point for every problem in which their answer reflected a halving or fractional complement understanding. A higher score on the screener meant the student demonstrated higher levels of unconventional understandings.

We evaluated the promise of this kind of screener with two studies. In the first study we evaluated how common these patterns of understanding were in a large sample (*n* = 390) of middle school students (i.e., grades 6-8; ages 11-14). Study 1 addressed the following research questions:

1. *Can unconventional fraction understandings (halving and fractional complement) be identified on a group administered written assessment?*
2. *What is the prevalence of these kinds of understandings?*
3. *Are unconventional understanding scores correlated with mathematics achievement scores?*

In the second study, we used this assessment to selectively recruit students to participate in an individual interview and assessment to determine whether students who demonstrated these unconventional understandings met rigorous DSM-5 dyscalculia criteria. Study 2 addressed the following research questions:

1. *Do students with high unconventionality scores on the Screener demonstrate the same unconventional understandings during a clinical interview?*
2. *Do these students with high unconventionality scores meet rigorous DSM-5 dyscalculia criteria?*

These studies together establish that building off the unconventional understandings identified in detailed analyses of extreme cases provides alternative avenues to selectively screen for characteristics of dyscalculia.

Study 1

In Study 1 we sought to evaluate whether it was possible to use the group administered Screener to identify the characteristic understandings found in students with dyscalculia. This paper-and-pencil assessment (see Appendix A) was administered to 390 students in grades 6-8 (i.e., middle school students, approximate age 11-14). Middle school (grades 6-8) was selected as the target age because these students would have had adequate exposure to fractions, given that fractions instruction generally begins in grade 3 in the United States (e.g., Common Core State Standards for Mathematics; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). We also collected state mandated achievement test mathematics scores to evaluate whether unconventionality scores were inversely correlated with achievement.

Methods

Data Collection

Mathematics teachers (*n* = 6) at a California middle school administered the Screener to all students during math class (*n* = 390). The teachers also provided each student's state mandated achievement test mathematics score from the prior academic year. In

January 2022, Volume 14, Issue 3, 243-267

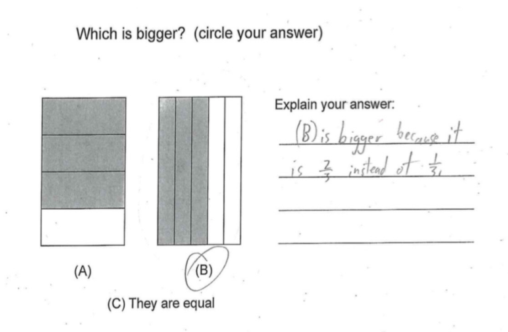
California, at the time, the mandated achievement test was the STAR test (Standardized Testing and Reporting program; http://www.cde.ca.gov/ta/tg/sr/). In order to collect these data and preserve student anonymity (a stipulation of our human subjects approval), when students completed the Screener, the teacher removed the cover page (with the student's name) and wrote the student's STAR test score on the now anonymized assessment. The research team received anonymized written responses on the Screener along with the student's STAR test score. One out of six of the teachers did not provide STAR test scores for her 50 students.

Analysis

Our research team scored the screeners for correctness and evidence of unconventional understandings. We assigned one unconventional understanding point for each answer which was consistent with an unconventional halving or fractional complement understanding (see examples Appendix A). For example, (see Figure 3) the student interpreted an area model of 3/5 and 3/4 as 2/3 and 1/3 (unshaded parts/shaded parts), respectively. The student was given one unconventional point for this problem because the student's response (which treated the unshaded pieces as focal) aligned with a fractional complement understanding.

Figure 3

Student work "(B) is bigger because it is 2/3 instead of 1/3." This answer would receive one unconventional understanding point for fractional complement because 3/4 and 3/5 were interpreted in terms of the fractional complements, 1 unshaded part for ¾ and 2 unshaded parts for 3/5, respectively (i.e., 1/3 and 2/3; unshaded/shaded)



Reliability and Validity Measures

All assessments were scored by at least two different scorers (see Appendix A for scoring criteria) Reliability for scoring was high: 97.9%. All discrepancies were resolved during our research meetings by reviewing the students' answers and our scoring criteria and reaching a consensus decision.

To evaluate the validity of this screener we conducted an item factor analysis. The parallel analysis showed that there is more than one factor measured by the test. The exploratory factor analysis further confirmed that a two-factor model outperformed a one factor model for these data. We determined the two factors were, as hypothesized: halving and fractional complement. Items 3 and 4 were removed from the confirmatory factor analysis because they were not associated with either fractional complement or halving. The confirmatory factor analysis showed that Items 1, 2, 5, and 12 loaded on factor 1 (halving), with questions 1 and 2 ("draw ½" and "draw another way to show ½") loading strongly on factor 1 (halving). The confirmatory factor analysis indicated that items 6, 7, 8, 9, 10, 11, 13 all strongly loaded onto factor 2 (fractional complement). The results of the confirmatory factor analysis are presented in Table 1, standardized factor loadings are between -1 and 1, with larger absolute values indicating a stronger association between the item and the factor. Because this screener is measuring two factors, Cronbach's alpha was understandably low (0.61), but the correlation between the two factors was moderately high (0.31).

Table 1

Standardized Loadings for 2-Factor Confirmatory Model of Unconventional Fraction Understandings (n = 390)

Item Number	Question Description	Factor 1 – Halving	Factor 2 – Fractional Complement
1	Draw ½	0.98	
2	Draw ½	0.99	
5	Interpret ½	0.35	
6	Compare 1/6 and 1/8		0.82
7	Compare 2/8 and 5/8		0.74
8	Interpret area model of 4/5		0.79
9	Interpret area model of 8/10		0.76
10	Compare ¾ and 3/5 area models		0.86
11	Compare 4/5 and 3/5 area models		0.90
12	½+1/4 =	0.60	
13	Interpret eight 1/10		0.53

Results

The results for Study 1 are presented in three parts. First, to evaluate whether it was possible to identify unconventional understandings on a written assessment, we present some exemplar written responses which illustrate unconventionality, either a fractional complement or a halving understanding. Second, we present an overview of the students' scores on the Screener to report the prevalence of these understandings. Finally, we evaluate whether unconventionality scores on the screener were associated with students' standardized mathematics achievement test performance.

Exemplar Unconventional Understandings

To illustrate prototypical unconventional understandings, we present several examples from students' responses and discuss how these reflect a potential halving or fractional complement unconventional understanding.

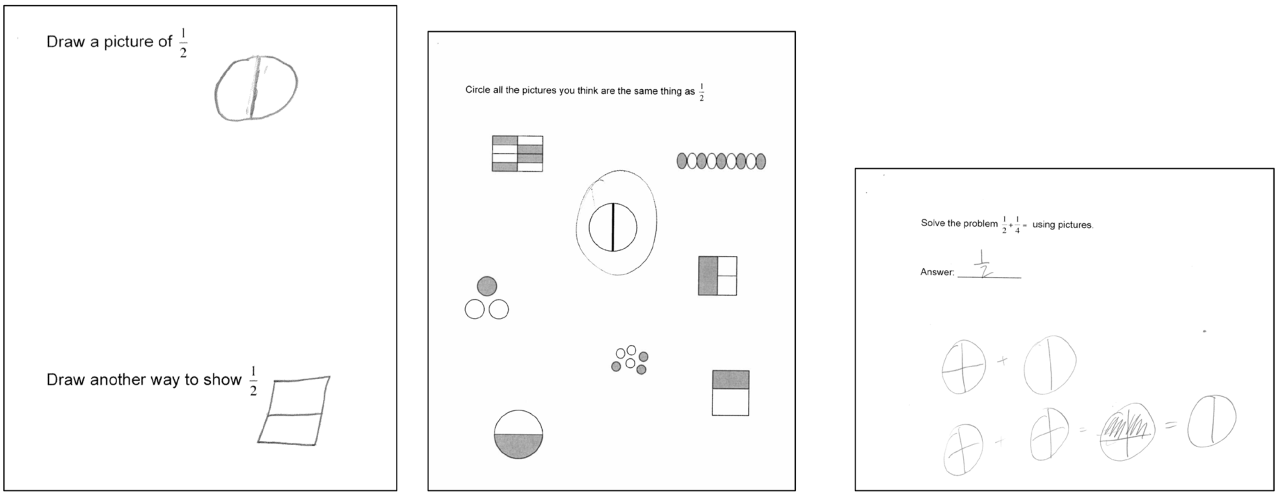
Unconventional Halving Understanding

Students' answers were coded as consistent with a halving understanding (Lewis, 2014) if they drew or interpreted the fraction ½ as a halved shape (see Figure 4). For example, a "halving understanding" was reflected in Figure 4a because the student drew a shape and partitioned it in two but did not shade or label either piece. Similarly, instances in which students selected an unshaded halved circle as a valid representation of ½ were considered consistent with a halving understanding (see Figure 4b). Finally, some students represented the fraction ½ without shading when asked to solve the problem ½ + ¼=. In this particular example (see Figure 4c), the student represented both ½ and ¼ with no shading. It is unclear what the student's intermediate drawings were intended to represent, but their final answer (an unshaded halved shape) was interpreted as ½ in their final answer and therefore was coded as consistent with a halving understanding.

Unconventional Fractional Complement Understanding

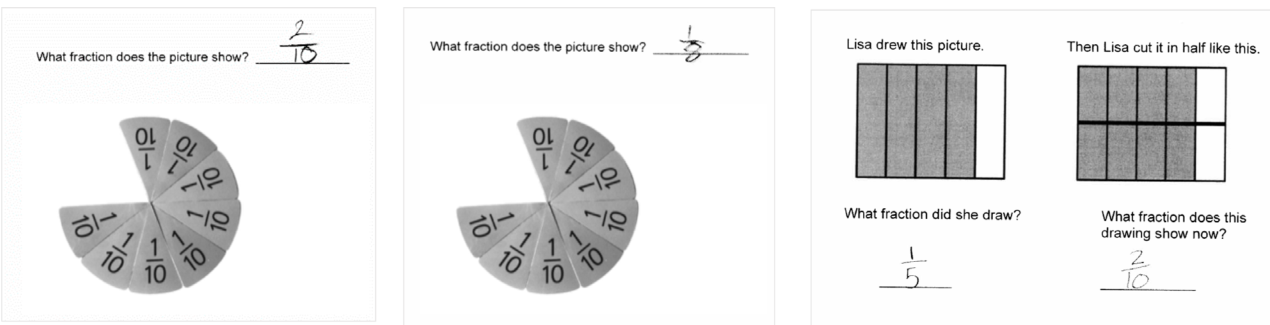
The fractional complement understanding occurred more often in cases in which the problem involved interpretation of a fraction. For example, student answers indicative of a fractional complement understanding included judging eight one-tenth pieces to be 2/10 (pieces missing/total pieces; see Figure 5a) or 1/8 (one empty space/pieces shown; see Figure 5b). Similarly, answers in which the student interpreted an area model in terms of the unshaded pieces (e.g., interpreting 4/5 as 1/5 (unshaded/total) and 8/10 as 2/10 (unshaded/total)) were also coded as indicative of a fractional complement understanding (see Figure 5c).

Figure 4
Exemplar written responses coded as consistent with a halving understanding



(a) (b) (c)

Figure 5
Example answers coded as consistent with a fractional complement understanding



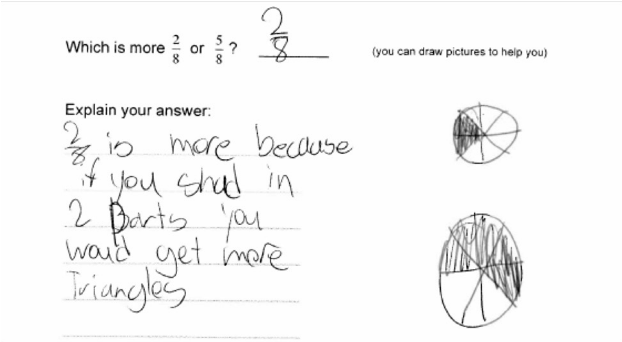
(a) (b) (c)

January 2022, Volume 14, Issue 3, 243-267

Fractional complement understanding was also evident in errors involving comparison of fractions. For example, a student incorrectly judged that 2/8 was larger than 5/8 explaining, "2/8 is more because if you shad in 2 parts you woud get more triangles" (see Figure 6). In this example, the student presumably sees more "triangles" in the drawing of 2/8 because there are 6 unshaded parts versus the 3 unshaded parts in the drawing of 5/8.

Figure 6

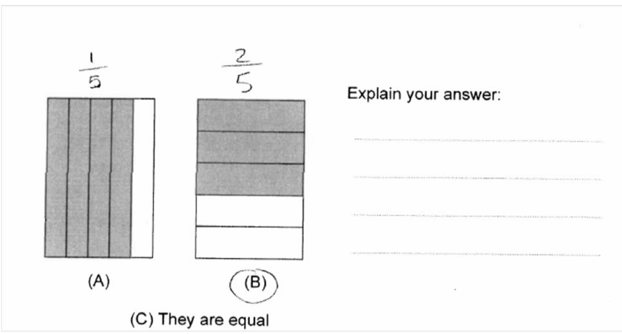
Student answer and explanation that 2/8 is more than 5/8



Similarly, when students made errors on comparing an area model of 4/5 and 3/5, their answers often reflected a fractional complement understanding. For example, one student interpreted the area model of 4/5 as 1/5 and the area model of 3/5 as 2/5 (see Figure 7). In both cases the student attended to the unshaded parts as the focal fractional quantity and therefore incorrectly determined that the latter was larger.

Figure 7

Student answer and explanation that an area model for 3/5 is larger than 4/5



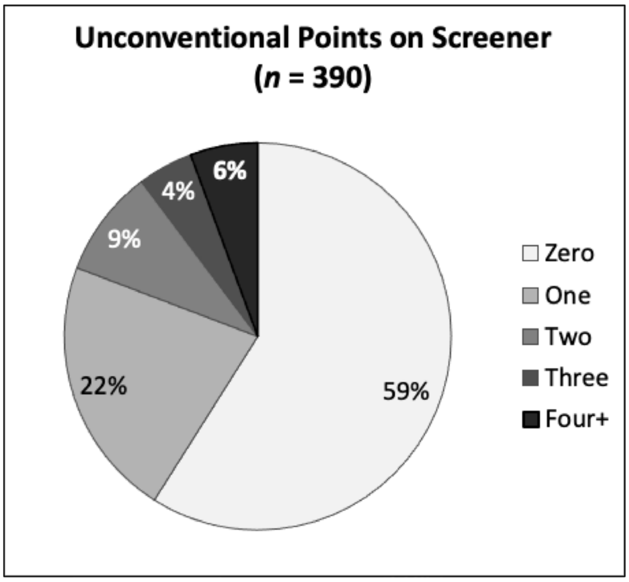
Although both halving and fractional complement are distinct understandings on this Screener we totaled the number of answers which were consistent with either a halving or a fractional complement understanding to produce one total score of unconventionality. A student receiving a higher unconventionality score would have more answers which indicated a fractional complement or halving understanding.

Student Performance on the Screener

To analyze the students' scores on the Screener, we considered the total unconventionality score obtained by each student. Only 6% of students had an unconventionality score of four or more points (indicating answers aligned with an unconventional understanding on more than 30% of problems; see Figure 8). The majority of students (59%) demonstrated no unconventional understandings. Another 22% of all students received only 1 unconventional point, and more than 63% of these students received an unconventionality point for circling the unshaded circle partitioned in 2 as one possible representation of ½ along with other valid representations. Therefore, as expected, most students demonstrated no unconventional understanding on the Screener.

Figure 8

Percentage of students who scored in each unconventional point range on the Screener (n = 390)



Achievement Test Scores

We collected student achievement scores to investigate whether unconventionality was simply a characteristic of low mathematics achievement (and consequently simply replicating achievement measures). We evaluated whether the students' unconventionality scores were correlated with their standardized achievement test scores. For this analysis we omitted 89 students for whom we did not receive STAR achievement mathematics test scores. One teacher did not provide this information to the research team (*n* = 50), and there were missing data for specific students in other classes. This missing data could be due to a variety of reasons, student's absence during STAR testing, transferring to the district or class, or an error of omission on the teacher's part.

When viewing the scores as a scatterplot (see Figure 9), it is evident that the students with the highest unconventionality scores were not necessarily the lowest achieving students, and some of the lowest achieving students had no unconventionality points. This suggests that this kind of approach – identifying characteristic patterns of reasoning – may be a promising approach to begin differentiating dyscalculia from low mathematics achievement due to other factors.

Figure 9

Scatterplot of achievement test scores and unconventionality points on the Screener, identical values are jittered

Summary and Conclusion

Study 1 found that the unconventional understandings documented in students with dyscalculia were evident on the group administered written Screener. This study suggests that these unconventional understandings, previously only documented with time intensive qualitative analysis of video data, are possible to identify in a group administered screener. Furthermore, the percentage of students with higher unconventionality scores (i.e., 4+ points) was approximately equivalent to the estimates for prevalence of dyscalculia (Shalev, 2007). Data from state mandated assessments suggested that high unconventionality scores were not only occurring in the lowest achieving students; furthermore, not all low achieving students demonstrated these unconventionalities. This suggests that this screener is measuring something different than low mathematics achievement. Due to the anonymized nature of the data we were not able to follow up with individual students who had high unconventionality scores. It remained an open question whether students who demonstrated high levels of unconventionality on the assessment would continue to exhibit these understandings over time and whether those students would also meet standard dyscalculia identification criteria. To investigate these questions, we conducted Study 2.

Study 2

In Study 2 we wanted to determine if the unconventional answers given on the Screener persisted and whether these students met rigorous DSM-5 dyscalculia criteria. We administered the Screener to 80 middle school students and recruited those students with high unconventionality scores to participate in an additional individualized assessment. The criteria for "high unconventionality" was set at four or more unconventional points, because this indicates reliance upon unconventional understandings across a significant number of problems (i.e., more than 30% of problems). Although it may have been interesting to assess students with two or more unconventional points to determine if they have an unconventional understanding of standard pedagogical representations, we focused on students with the highest levels of unconventionality (4 or more points) due to time constraints. We conducted individual problem solving clinical interviews to evaluate whether these students did rely upon unconventional understandings. We conducted an individualized standardized achievement test (Woodcock Johnson IV; Schrank et al., 2014) and background interview to determine whether these students with high unconventionality scores met standard DSM-5 dyscalculia criteria.

Methods

Data Collection

All middle school students (grades 6-8) enrolled at a private school for students with language-based learning disabilities were assessed using the Screener (*n* = 80). The student's enrollment at this school ensured that these students had intelligence scores in the normal range and therefore eliminated the possibility of intellectual disability. We anticipated that a higher percentage of students recruited from this school would have high unconventionality scores given the documented comorbidity between dyscalculia and dyslexia (e.g., Knopik et al., 1997; Wilson et al., 2015). However, Lyon, Shaywitz and Shaywitz (2003) argue that although there is well known comorbidity, the cognitive characteristics associated with each of these disabilities are sufficiently distinct (e.g., phonemic awareness vs. number processing) and do not present a problem in studying one independent from the other. In addition, the reading demands of the screener were minimal, and therefore, the impact of the student's difficulties with reading were not considered to be problematic for this study.

Each mathematics teacher at the school administered the Screener to their students (*n* = 80). The cover page and first page of the assessment were numbered with a test ID. When students completed the assessment, the teacher removed the cover page (with the student's name), and retained the cover sheet for subsequent recruitment efforts. The research team scored these assessments anonymously. To recruit students for the main study, the teachers were given a list of test IDs associated with students who had unconventionality scores of at least 4 points. Teachers used the cover sheets to distribute consent materials to students who qualified. Consents were directly returned to the research team through the U.S. Postal Service. Parents and students who did not want to participate were asked to simply discard their forms to preserve their anonymity. Seven students met the high unconventionality threshold and we received consent forms for three of these students.

Several kinds of data were collected for the three students who participated in the individual assessment including: (a) background interview, in which the students reported on their resources and their prior experiences learning and doing mathematics, (b) a clinical interview problem solving session in which the student solved the questions from the Screener, and (c) an individually administered standardized achievement test. Due to scheduling constraints these individual sessions were conducted eight months after the original assessment data.

Background interview

The students were interviewed and asked to provide a self-report of their academic background, the kinds of difficulties they experienced in mathematics, their level of effort, available resources (e.g., tutoring, teacher help), and home language (see Appendix B). The goal of the background interview was to assess the student's level of perceived effort and educational resources as well as to establish rapport. Note that we did not collect data on the socioeconomic status of the student and their families, but these students were all paying tuition to attend a private school, suggesting the families had sufficient financial resources.

Problem solving interview

In the problem solving clinical interview, the students were asked each of the questions from the Screener. For each of the student's answers, the interviewer asked the student to explain their solution and/or process. Because it had been over eight months between the administration of the Screener and the interview, we were not concerned about practice effects.

Both the background interview and problem solving interview were video recorded and were conducted by the first and second authors.

Standardized measure. To determine if the students met the low mathematics achievement clinical criteria established in the DSM-5, all three students were assessed using the mathematics subtests of the Woodcock Johnson IV Test of Achievement (Schrank et al., 2014). The subtests included, Applied Problems, Calculation, and Math Facts Fluency.

Analysis

Screener

As in Study 1, the written screener assessments were scored by at least two different scorers (see Appendix A for scoring criteria). Reliability for scoring was high, 97.6%. All discrepancies were resolved during our research meetings by reviewing the students' answers and reaching a consensus decision.

Case study analysis

For the three students who qualified for and consented to participate in the individual assessment, we transcribed the video recordings and scanned all written artifacts.

Background interview

For the background interview, the first and second authors reviewed the students' answers and identified any potential confounding factors which could explain the student's mathematics difficulties. We looked for self-reports of insufficient educational opportunity, insufficient resources, poor prior teaching, or difficulty with attention or behavioral control.

Problem solving interview

For the problem solving interview, the first and second authors coded these videos using the coding scheme from the Screener. We allowed student's explanations to disambiguate answers when needed, similar to the way we used written explanations on the Screener. Reliability for this coding was 85.2%. All discrepancies were resolved by reviewing the video and reaching consensus on how the question should be scored.

Results

The results are presented in two parts. First, we present each case by illustrating the students' unconventional answers from the screener and how these same patterns of reasoning were evident during the interview. Then we evaluate whether these three students met the standard DSM-5 dyscalculia criteria.

Case Study Students with High Unconventionality Scores

Out of the 80 students assessed, only 7 students (9%) had an unconventionality score of four or above. These 7 students were recruited to participate in the interviews and standardized assessment. Three students, "Ryan," "Lily," and "Maddie" (all pseudonyms) returned consent forms. All three students who consented to participate in the main study demonstrated the same unconventional understandings during the interview that they did on the screener (see Table 2). Ryan and Lily demonstrated halving and fractional complement understandings on both the screener and interview. Maddie demonstrated a fractional complement understanding, and did so on both the screener and interview. For each case study student, we present answers given on both the screener and the interview which highlight the persistence of these unconventional understandings.

Table 2

Unconventional understanding points on the screener and interview

	Assessment	Fractional Complement Points	Halving Points	Total Unconventional Understanding Points
Ryan	Screener	3	1	4
	Interview	1	3	4
Lily	Screener	3	1	4
	Interview	6	1	7
Maddie	Screener	4	0	4
	Interview	5	0	5

Ryan

On the screener Ryan demonstrated both an unconventional halving and fractional complement understanding. In Ryan's answers on the screener, a halving understanding was evident on one problem, in his selection of the non-shaded halved circle as a valid representation of ½. Ryan also demonstrated a fractional complement understanding in his comparison of fractions on the screener. When asked to compare fractions, he incorrectly judged 1/8 to be greater than 1/6 and 2/8 to be greater than 5/8 drawing accurate areas models for each. He also incorrectly judged an area model for 3/5 to be greater than 3/4, and an area model of 3/5 to be greater than 4/5. In each instance, his explanations identified "more space" in the fraction he judged to be larger, which was consistently the fraction with more unshaded parts. This suggests that, particularly on comparison problems, Ryan was relying upon a fractional complement understanding.

On the interview both these unconventional understandings resurfaced but with different frequency. A halving understanding occurred more frequently, and fractional complement understanding occurred less frequently. When Ryan was asked to draw the fraction 1/2, he drew several different representations including a pizza, a pie, and a pedestrian "don't walk" sign (see Figure 10). In each of these cases, he omitted shading. When asked to identify the part of his picture that was one-half he indicated that one-half was the partition line.

Figure 10

Ryan's drawings of 1/2 (pedestrian "don't walk" sign, pizza, and pie)

Interviewer: Can you explain to me how your pictures show one-half?

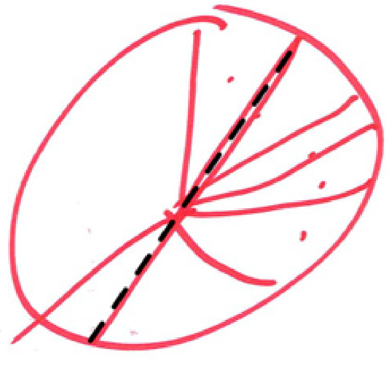
Ryan: Um, because they have a line right down the middle [points to line in the center of the pie, see Figure 10], and this side's equal [points to right side of pie], and this side's equal [points to left side of pie]. Like 1, 2 [writes ½] or... [starts pointing to the pizza slices in his drawing] I don't know how many pieces of pizza that is, but, yeah.

Interviewer: So where is the one-half in this picture? [points to pizza]

Ryan: [points along center dividing line; see Figure 11] Right there.

Figure 11

Ryan's drawing of 1/2 of a pizza with a dotted line indicating where he gestured to identify where one-half was in his drawing



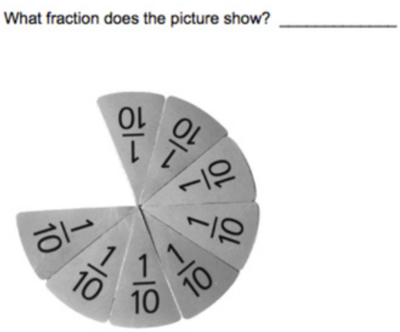
January 2022, Volume 14, Issue 3, 243-267

In Ryan's explanations he focused on the equality of the two halves and the partition line itself. Although Ryan's drawing of the pizza pieces and his attempt to count them up, suggests that he might have been attending to one-half of his circle (or pizza), when specifically asked where the one-half was in his picture, he identified the partition line itself and not the pieces on one side of the pizza as the representation of 1/2. Ryan's unshaded and halved representations along with his explanations focusing on the partition line itself was taken as evidence of his halving understanding.

In contrast to Ryan's halving understanding, a fractional complement understanding occurred only once during his interview. On an interpretation problem, Ryan determined that the eight 1/10 pieces (see Figure 12) was equal to 1/8. This reflected a fractional complement understanding because he attended to the missing part (perceived as 1 missing part) and the number of pieces displayed (i.e., 8). This was coded as a fractional complement understanding because it involves naming the fraction in terms of the missing amount.

Figure 12

Interpretation problem which presents eight 1/10 pieces and asks student to interpret the amount shown



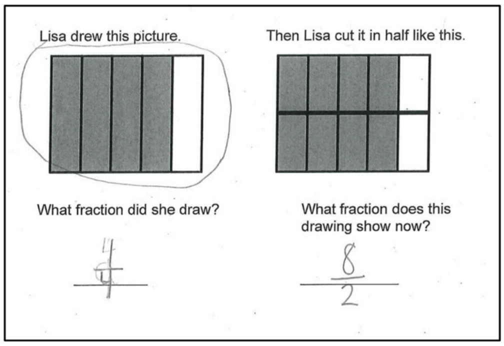
Although the halving and fractional complement understandings were evident on different problems and had different frequencies on the screener and in the interview, in both instances, Ryan's answers and explanations indicated his reliance upon these understandings found in students with dyscalculia.

Lily

Lily demonstrated both a halving and fractional complement understanding on the screener and interview. Like Ryan, Lily selected the unshaded halved circle as a valid representation of ½, and did so both on the screener and interview. Therefore, there was consistency in her halving understanding. Lily also demonstrated consistency in her fractional complement understanding. On the screener Lily interpreted 4/5 and 8/10 as 1/4 (unshaded/shaded) and 8/2 (shaded/unshaded), clearly attending to the unshaded pieces as focal (see Figure 13). In addition, many of her area model comparison problems were also aligned with attending to the fractional complement (e.g., larger fraction determined by largest unshaded area; Figure 14), but these were not coded as such because she did not provide a written explanation for her judgments.

Figure 13

Lily's screener responses that were coded as consistent with a fractional complement understanding, because she focused on the unshaded (fractional complement) pieces in her interpretation of the fraction



Lily's interpretation of the area models 4/5 and 8/10 during the interview was similar to her answers on the screener. During Lily's interview, she again identified 4/5 as 1/4 (unshaded/shaded) and identified 8/10 as 2/8 (unshaded/shaded), focusing on the pieces she referred to as "left."

Figure 14

Lily's answers that were potentially due to a fractional complement understand (judging fractions based on unshaded parts) but were not coded as unconventional, because she did not provide an explanation for her answer

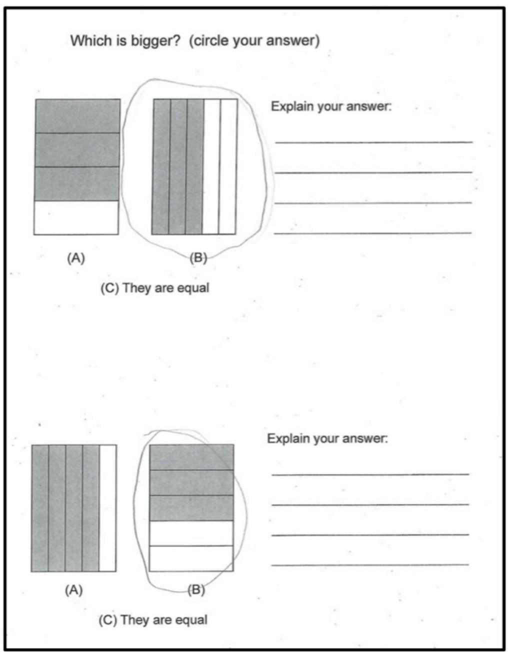
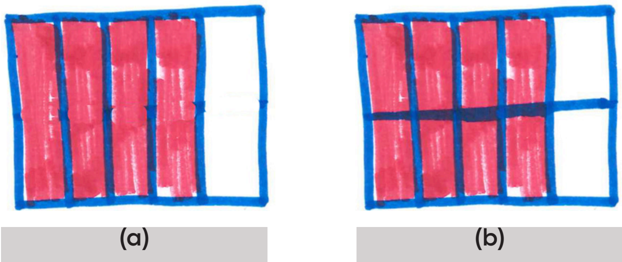


Figure 15
Tutor drawn representation of 4/5 (digitally recreated), which was repartitioned to produce 8/10



Interviewer: [draws 4/5; see Figure 15a] Okay, so this is a picture of –

Lily: One-fourth.

Interviewer: So this is a picture of one-fourth?

Lily: Yeah.

Interviewer: Okay. So then another student came along and did this to her picture. [draws horizontal line; see Figure 15b] Can you tell me what fraction that is?

Lily: [pointing to unshaded sections] Is she crossing out this? Oh.

Interviewer: So she...

Lily: Two-eighths.

Interviewer: Two-eighths?

Lily: Yeah.

Interviewer: Okay. Can you tell me how you got that answer?

Lily: Well, [points to picture], if you divide it in half, this makes 8, because 1, 2, 3, 4, 5, 6, 7, 8, [gestures over 8 shaded pieces, each in turn] and then there's 2 left over [points to 2 unshaded pieces].

Lily interpreted the fraction in terms of the unshaded pieces and referred to those pieces as "left." Lily's tendency to interpret fractions by attending to the fractional complement (unshaded parts) also emerged as she compared area models of 3/4 and 3/5. As she had done on the screener, she judged the drawing of 3/5 to be larger. When asked to explain her answer, she interpreted each fraction in terms of the number of unshaded parts and shaded parts; 3/5 was interpreted as 2/3 and 3/4 was interpreted as 1/3.

Interviewer: In looking at these two pictures, can you tell me which one is larger, or are they equal?

Lily: [touches drawing of 3/5 firmly with finger, 5 times; see Figure 16] This one.

Interviewer: Do you want to circle it?

Lily: Naw, that's okay. Just that one [points to drawing of 3/5].

Interviewer: Can you tell me – you're pointing to this one –

Lily: Yeah.

Interviewer: – it's larger? Can you tell me how you know that?

Lily: [points to drawing of 3/5] There's... it's two-thirds, and then this one is [pointing to drawing of 3/4], one-third. So this one's more [points to drawing of 3/5], there's 2 that got left out kind of.

Lily's judgment that 3/5 was larger than 3/4 was based on her attention to the unshaded pieces, which she again referred to as "left out." Lily consistently relied upon a fractional complement understanding. Given the consistency of Lily's answers on both the screener and the interview, the fractional complement understanding provides a plausible explanation for Lily's errors on the area model comparison problems on the screener (see Figure 14).

Figure 16
Printed question asking student to compare ¾ and 3/5 represented with area models

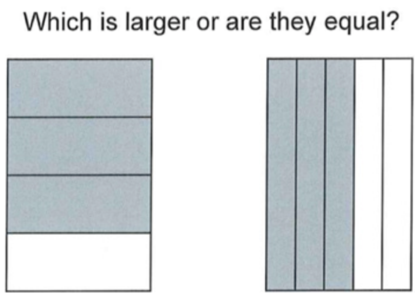
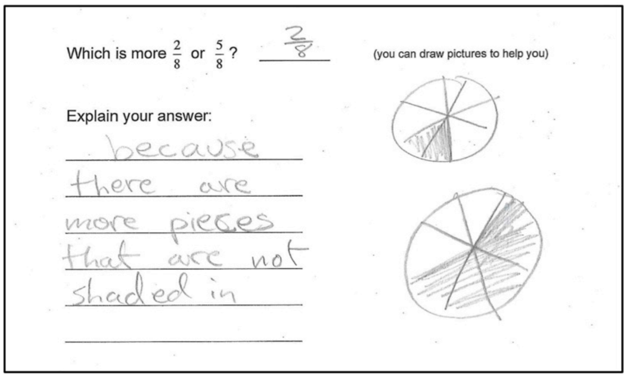


Figure 17
Maddie's written responses on the screener for the comparison problem of 2/8 and 5/8, in which she determined 2/8 was larger



Maddie

Unlike Ryan and Lily, there were no instances of Maddie demonstrating a halving understanding on either the screener or the interview. She did however demonstrate a fractional complement understanding on both. When asked to determine which quantity was more, she struggled particularly when the denominators of the fractions were the same. For example, she judged 2/8 to be larger than 5/8. Her solution helps illustrate how a fractional complement understanding was evident in this problem and how it was problematic (see Figure 17). Maddie drew canonical representations for both 2/8 and 5/8, using shading to represent the fractional quantity. However,

January 2022, Volume 14, Issue 3, 243-267

she then judged 2/8 to be larger because there were "more pieces not shaded in." This highlights the disconnection between her canonical use of shading in her construction of the area models and her unconventional focus on the unshaded parts in interpreting her own drawings. The quantities she compared were not the quantities she herself drew, but the fractional complements.

Maddie again attended to the unshaded pieces when asked to compare area models of 4/5 and 3/5 (see Figure 18), incorrectly judging that 3/5 was larger because there were more parts not colored in. For both same denominator comparison problems, she incorrectly believed the smaller amount was larger, and in each case, she justified her answer by identifying that there was more that was not shaded.

In addition to these comparison problems, Maddie's fractional complement understanding was also evident when she interpreted the eight 1/10 pieces as 2/8 (pieces missing/pieces shown; see Figure 19).

Although it was not as evident during the interview, Maddie continued to rely on a fractional complement understanding. When asked to interpret a drawn area model of 4/5 (see Figure 20a), she, like Lily, interpreted it first in terms of the unshaded amount (1/4; unshaded/shaded). When asked to justify her answer of 1/4, she justified it by noting the number of boxes colored in, but did not change her answer. When the interviewer repartitioned this area model to produce 8/10 (see Figure 20b), she again initially focused on the two unshaded pieces. Unlike her previous answer, she eventually corrected this error. Throughout her explanations she vacillated between different interpretations of the representation. First providing a fractional complement answer (1/4) and justifying her answer with the shaded region, and then correcting her final interpretation (8/10) and justifying it based on the fractional complement.

Figure 18

Maddie's written responses on the screener on a comparison problem of 4/5 and 3/5 in which she was asked to circle the larger amount. She explains that 3/5 is larger because "there are two lines that are not colered in."

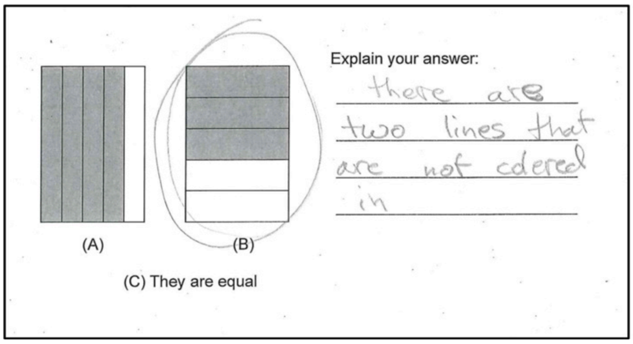


Figure 19

Maddie's written work interpreting eight 1/10 pieces in terms of the number of pieces missing (2) over the number of pieces shown (8)

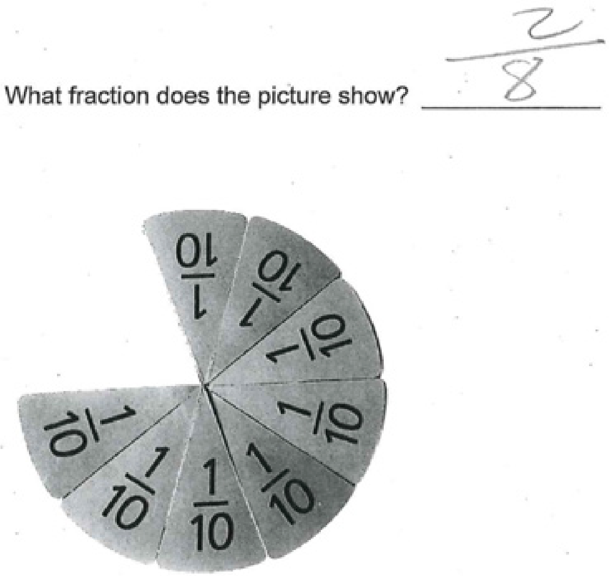
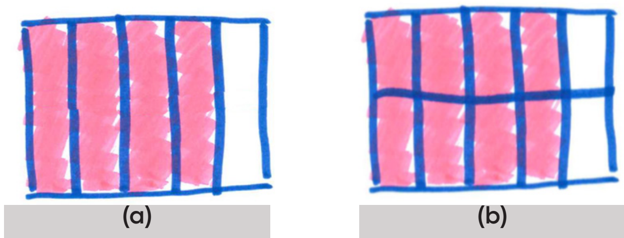


Figure 20

Tutor drawn representation of 4/5 (digitally recreated), which was then repartitioned to create 8/10



Interviewer: Okay, one student I was working with drew a picture like this. [draws rectangle with 5 sections, colors in 4; see Figure 20a] What would you say that's a picture of?

Maddie: I think that would be one-fourth.

Interviewer: How do you know?

Maddie: Because um, 4 – 4, I mean, um, 4 out of 5 boxes were colored in.

Interviewer: Okay, 4 out of 5 boxes were colored in. So then another student came along and cut it in half like that. [draws line down the middle; see Figure 20b]

Maddie: Um, that would be...

Interviewer: What would you say that is now?

Maddie: It would be 8 out of 2 – or, 2 out of 8. No, 4 out of 8. Wait. 8 out of 10. 8 out of 10.

Interviewer: 8 out of 10? How do you know?

Maddie: Because um, now that the squares are cut up, [touches picture], there are 8 that are colored and 2 that are left.

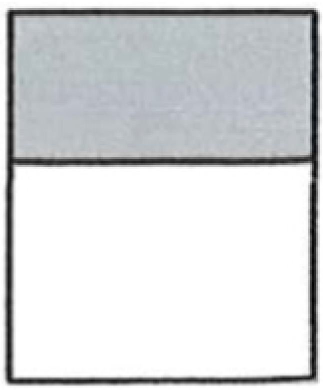
In her interpretation of 8/10, Maddie corrected her initial fractional complement answers (8/2 shaded/unshaded and 2/8 unshaded/shaded) and correctly determined that the repartitioned fraction was a representation of 8/10. However, she still attended to the fractional complement (2 pieces) and referred to them as "left." – one of the defining characteristics of the fractional complement understanding.

Maddie's focus on the unshaded space as the fractional quantity was also evident when asked to justify why she (correctly) did not select the unequally partitioned area model as a valid representation of ½ (see Figure 21). When asked why she did not select it, she interpreted the white (unshaded) part as the focal fractional quantity, and judged that the area model was more than ½.

> *Interviewer: Can you explain why you didn't choose this one?*
>
> *Maddie: Because the white has more of – the white is covering more of the square.*
>
> *Interviewer: So is this going to be less than one-half or more than one-half?*
>
> *Maddie: Um... [pause] I think it would be... [pause] I think it would be more. Um, because the white has more.*

Figure 21

Printed image that Maddie determined was more than 1/2



In her justification, Maddie understood this representation to be more than ½, suggesting that she was attending to the white space as the focal fractional quantity.

As in Ryan's case, there was some variation on the specific problems, which elicited her fractional complement understanding. On the screener it was primarily on comparison problems, and in the interview, it was primarily during interpretation problems. These data suggest that Maddie relied upon a fractional complement understanding to make sense of various fraction representations in various contexts.

Summary

All three students demonstrated unconventional understandings during the interview that were consistent with those documented in adults with dyscalculia (Lewis, 2014). Although there were often differences in the specific problems in which the understandings emerged, there was consistency in the nature of the understandings themselves. Maddie relied on a fractional complement understanding, and did so on both the screener and interview. Ryan and Lily demonstrated both a fractional complement and halving understanding. In Lily's case there was consistency in the problems and specific reasoning on the screener and interview, whereas in Ryan's case the same understanding persisted but with different frequencies and on different problems. We judge the screener to be a useful tool to identify students with these characteristic unconventional understandings given their high unconventionality scores on both the screener and interview. We then evaluated whether these three students met the standard criteria for dyscalculia classification established by the DSM-5.

Dyscalculia Classification

The DSM-5 requires that students with dyscalculia have persistent difficulties in mathematics that are evident during formal schooling and result in below average achievement. The DSM-5 recommends operationalizing "below average" as 1.5 standard deviations below the population mean on a norm referenced achievement test, which corresponds to the 7th percentile. Additionally, the student's low achievement must not be due to lack of educational opportunity, poor instruction, lack of fluency in instructional language, developmental delay, or a sensory, motor, or neurological disorder.

In order to evaluate whether these students also met the DSM-5 criteria for dyscalculia classification we considered students' composite and subtest scores on the Woodcock-Johnson Test of Achievement IV (WJ-IV) and self-reports of their educational history and opportunity. The WJ-IV scores for each student are presented in Table 3. Lily and Ryan clearly met the "below average achievement" criterion, as all of their subtests and composite scores were below the 7th percentile. Maddie's percentile scores were more variable. Maddie met the below average achievement criterion on only one subtest – Math Facts Fluency – and in one composite score (Mathematics Calculation Skills). Math Facts Fluency is the only timed math assessment within the WJ-IV, and researchers have argued for the importance of timed assessments of mathematics performance to accurately identify students with dyscalculia (e.g., Berch, 2005; Mazzocco, 2009). Indeed, when completing the untimed sections, Maddie's progress through the questions was laborious and time intensive. This suggests that she may have developed ways of compensating for her difficulties (see Lewis & Lynn, 2018 for a discussion), but that her difficulties were more evident under time constraints. Because Maddie's score on a timed assessment fell below the 7th percentile, we argue that she meets the dyscalculia criteria based on this more sensitive measure.

Table 3

Percentile scores on the Woodcock Johnson IV Test of Achievement for the case study students.

	Ryan	Lily	Maddie
Mathematics Composite	1	0.2	24
Broad Mathematics	<0.1	<0.1	8
Math Calculation Skills	<0.1	<0.1	7
Applied Problems	7	2	29
Calculation	1	0.1	25
Math Facts Fluency	<0.1	0.2	2

In addition to the below average achievement criterion, the students' self-reports indicate that these difficulties were evident in early school years, and the difficulties were not attributable to a global developmental delay, hearing, vision, neurological, or motor disorder. All students were White native English speakers (see Table 4) and therefore entered the school context with considerable privilege. Based on the individual self-reports all students had sufficient familial and educational resources (e.g., homework club, individual teacher/parent help), decreasing the likelihood that environmental or social circumstances were the origin of their difficulties in mathematics. These students were attending a private school for students with language-based learning disabilities, and although it is possible that their difficulties with language impacted their ability to learn mathematics, none of the students identified reading difficulties as an issue for them in mathematics.

Table 4

Demographic information for case study students.

	Ryan	Lily	Maddie
Gender	Male	Female	Female
Race	White	White	White
Age (years-months)	13-11	13-2	13-9
Grade	8	8	8

Conclusion

All three students who demonstrated high levels of unconventionality on the Screener continued to demonstrate these same unconventional understandings on the interview. This suggests that these understandings do persist over time and continue to lead to specific kinds of answers. All three students also met the qualifications for the DSM-5 dyscalculia criteria. This suggests that it may be possible to screen for characteristics of dyscalculia with a group administered screener.

Discussion

These two studies together provide a proof-of-concept for a novel approach to addressing the intractable identification issues facing dyscalculia researchers. Through these studies we provided a model for leveraging case study work in powerful ways to go beyond the individual cases and consider the prevalence of these patterns of understanding more broadly. By using detailed qualitative studies of extreme cases to design group administered written assessments, it may be possible to make considerable progress towards delineating the unique characteristics of this disability. This kind of approach is novel in that it attempts to define and identify dyscalculia by the unique characteristics (i.e., unconventional understandings) rather than defining dyscalculia as performance deficits.

Study 1 demonstrated that the unconventional understandings documented in Lewis (2014) were atypical. Only 6% of middle school students had high unconventionality scores. The percentage of students with high unconventionality scores was approximately equal to the estimated prevalence of dyscalculia in the general population (Shalev, 2007). The fact that (a) not all low achieving students demonstrated unconventionalities, and (b) that the students with the highest levels of unconventionality were not necessarily the lowest achieving students, suggests that the Screener identified qualitative differences in understanding, rather than simply low achievement.

Study 2 helped establish the validity of the Screener for identifying unconventional understandings. The students with high unconventionality scores on the Screener in study 2, did rely upon and demonstrate unconventional understandings in their interviews. Furthermore, additional assessments found that all three of these students met rigorous dyscalculia criteria established by the DSM-5. These studies together provide evidence that it may be possible to build off characteristic understandings documented in adults with dyscalculia to develop novel approaches for identification. Unlike standard approaches which struggle to differentiate dyscalculia from low achievement, these studies suggest that it may be possible to identify the characteristics of dyscalculia on a group-administered assessment.

Evaluation of the Screener

The validity of this Screening assessment was also evaluated through item factor analysis, which confirmed that this assessment measured two factors: halving and fractional complement. Although there was variability in how strongly particular items loaded onto the associated factor, we find analytic utility in all items. For example, although items 3 and 4 (draw 3/5; draw 1 5/8) did not load onto fractional complement, these questions did provide essential information for how the student understood the shading when drawing area models. If a student used shading to represent the numerator (i.e., fractional quantity) in their drawings, but used the unshaded parts to interpret the fractional quantity, it suggests an unconventional understanding of the shading. It is precisely because of the disconnection between how students draw and interpret area models that these items would not load strongly onto fractional complement, but nevertheless provide important information about the students' understanding. Similarly, although item 5 (interpret ½) did not load as strongly onto factor 1 (halving) we believe that this item provides important insight. For example, it was only on this item on the Screener that Ryan's tendency to understand ½ as halving was evident. The interview demonstrated that Ryan did rely upon a halving understanding when he drew non-shaded halves and identified the partition line itself as a representation of ½. Therefore, although some items did not load strongly onto the two factors, we believe they provide important insight into the students' understanding.

Future Research

We acknowledge that this Screener only includes a small subset of ways in which students with dyscalculia may understand mathematics in different ways. It is possible that additional research into how these students represent these fraction quantities on the number line (Schneider & Siegler, 2010) or compare fraction magnitudes (Meert et al., 2009) would yield insight into their understanding of fraction quantity. The field needs to invest in more detailed studies of extreme cases to specifically identify the characteristics of this disability across a range of mathematics topics. This suggests a dramatic shift from a focus on identifying performance deficits in speed and accuracy, to a focus on identifying what students with dyscalculia are doing and how these understandings may be unconventional. Until then, leveraging these characteristics may enable the development of alternative identification approaches. For example, if dyscalculia impacts students' learning across all mathematics topics (e.g., Lewis & Lynn, 2018) it may be possible to selectively recruit students with unconventional fraction understandings and then explore how these students make sense of other topics, like algebra.

Implications for Research and Practice

The issue of accurate dyscalculia identification has far reaching consequences for research and practice. Current use of the low achievement criteria has resulted in heterogeneous groups of students erroneously labeled as dyscalculic. Studies of dyscalculia that rely on this problematic and imprecise proxy are often studying low mathematics achievement – often due to inequitable educational opportunities – in the name of dyscalculia. The unintended consequences of this widespread use of this insufficient operational definition has resulted in myriad studies arguing that students with dyscalculia simply lag behind their peers (e.g., Gonzalez & Espinel, 2002; Keeler & Swanson, 2001; Mabbott & Bisanz, 2008). Because low achievement is used as the sole criteria for dyscalculia classification, studies have argued that students with dyscalculia are simply delayed in their mathematical development, rather than qualitatively different (Geary & Hoard, 2005). The developmental lag theory suggests the same teaching methods should be effective and these students simply require additional time and exposure to standard instruction. Because this research is largely based on studies which have not employed a sufficient exclusionary definition to determine that the low achievement is due to a disability rather than social or environmental factors (Lewis & Fisher, 2016), we take issue with this theory and its resulting implications for instruction.

In our studies we contribute to the growing body of work that suggests that qualitative differences in performance may be a productive approach to differentiate students with dyscalculia from students with low achievement due to other factors (e.g., Desoete & Roeyers, 2005; Mazzocco et al., 2008; 2013; Mazzocco & Devlin 2008). This suggests that a "more of the same" instructional approach will not work for these learners, because they have difficulties that are qualitatively different than their peers. We argue that the unconventional understandings identified in the Screener and Interview impact a student's ability to access standard instruction and these students may require different kinds of instruction that takes these issues of access into account (Lewis, 2017). At the heart of both unconventional understandings is a tendency to understand representations of quantities as representations of action (e.g., taking or halving). Students who rely upon these kinds of qualitatively different unconventional understandings require alternative forms of instruction that acknowledge and build upon these students' unique resources (Lewis, 2017).

If used in practice, this Screener should just be used as a first step in a holistic evaluation of the student. All students may experience unconventional understandings when first learning how to use and translate between different mathematical representations (symbols, language, and pictorial; Viseu et al., 2021), so this Screener may not be effective with younger students first learning about fractions. For students with adequate opportunity to learn about fractions, persistent evidence of unconventional understandings may signify an issue of access. For students with suspected dyscalculia, multiple assessments including observation, interview, and other nonstandard assessment are recommended to determine if the difficulties are due to dyscalculia or other factors (Mundia, 2017). These kinds of nonstandardized assessments help educators identify unconventional understandings, issues of access, and suggest how to design alternative accessible instruction for that student (e.g., Lewis, 2017).

Limitations

There are several limitations of the current study. First, this assessment was limited to exploring basic representation and interpretation of fraction quantities, which represent a narrow slice of fraction concepts and skills. Although some researchers might argue that the narrow topic domain is problematic because mathematics is componential in nature (Dowker, 2015), we argue that these unconventional fraction understandings are indicative of underlying number processing issues, representing quantities as actions, rather than objects (Sfard, 1991). We do not claim that the Screener captures the myriad ways in which dyscalculia may manifest, however, students who have been identified using this screener have had similar unconventional understandings when working with integers (Lewis et al., 2020) and algebra (Lewis et al., 2022), suggesting the utility of identifying these kinds of unconventional understandings even in a narrow topic domain. We do not propose the Screener to be a test for dyscalculia, instead these studies are intended to illustrate the potential utility of a general approach to drawing upon evidence of unconventional understandings identified in detailed analyses of extreme cases to design more sensitive screening tools.

A second limitation of this study is that in study 2 the dyscalculia criteria were assessed only for students who were attending a school for students with language-based learning disabilities. It is possible that the students' language-based learning disability did impact their understanding of mathematics. There is specific academic language associated with fractions (e.g., numerator, denominator; Bossé et al., 2019), and it is possible this created an additional barrier for students. We cannot fully address issues of comorbidity that this participant population raises. However, in other preliminary work, there is some evidence that the Screener works to identify college-aged students with dyscalculia with no other learning disabilities (Lewis et al., 2020). Future work should consider whether this kind of screener has utility for identifying students without other learning disabilities in a general population of students.

Third, although we documented unconventional understandings in the case study students, it is an open question what kind of instruction would be necessary to support their understanding of fractions as quantities. Although research has demonstrated this kind of re-mediation with one of the adult students from the first case study (Lewis, 2017), more research is needed to determine if similar approaches would be effective for younger students.

One final limitation, is that due to the nature of the anonymous data collection for Study 1, we relied upon the teachers recording of test scores on the written assessments. These are the only data that were not double coded, and therefore, inadvertent errors could have been made. Because this was an ancillary point and not the main objective of the study, this potential for error in the data was not seen to be critical.

Conclusion

These studies established a proof-of-concept for designing a group administered screener by leveraging the qualitative differences identified in students with dyscalculia. This provides a novel approach to address the long-standing methodological issues facing the field with regards to identification and classification of students with dyscalculia. We believe that conceptualizing dyscalculia in terms of developmental difference rather than deficit has the potential to greatly impact both research and practice for students with dyscalculia. The screener identified students who understood standard tools for representing fractions (drawings, symbols) in ways that were unconventional and would render these standard mediational tools inaccessible. This suggests that instruction which relies on these standard representations would be inaccessible and that alternative more accessible instruction may need to be designed. Students who score high on this screener are worthy of further assessment to evaluate how to support their fractions learning and to determine if they have other issues of access across other topic domains.

[1]The terms "dyscalculia" and "mathematics learning disability" are used interchangeably in the field (Mazzocco, 2007). We use the former because this term is more commonly used internationally. We differentiate dyscalculia – which involves a difference in how the student processes numerical information – from students with mathematics learning difficulties who may have low achievement in mathematics due to a variety of social or environmental causes.

[2]Response-to-intervention approaches, which are sometimes used in schools to identify students who qualify for special education services, are not often used in research on dyscalculia because they lack specificity and methodological rigor. A small number of studies (2%, based on a systematic literature review; Lewis & Fisher, 2016) have used growth curve analysis

to identify students who not only are low achieving, but also have slow growth, however this kind of Response-to-Intervention approach is not commonly used in the field.

[3]It is worth noting that this pathologizing of human variation can be thought of as problematic, and this delineation of humans into "normal" and "abnormal" has its origins in the eugenics movement (e.g., Davis, 2006). The point here is not to take a position on whether the category of dyscalculia is morally, ethically, practically, or politically appropriate, but to identify that when disability categories have been defined, it has often started with the close and careful clinical appraisal of individuals considered to be exceptional. In this study our goal is not to further pathologize human variation, but to better understand how cognitive differences may result in inaccessibility in mathematics. By improving identification approaches we hope to (a) enable students with this disability to advocate and obtain access to accommodations to address the inaccessible mathematics context and (b) avoid inappropriately labeling students with low mathematics achievement as disabled.

[4]Although the fractional complement for 3/4 is 1/4, we also classified instances where the student interpreted the fraction as unshaded/shaded (e.g., 1/3), because their answer suggested that the student was attending to the fractional complement (the one unshaded part) as the focal quantity.

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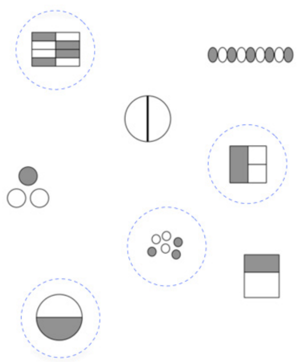
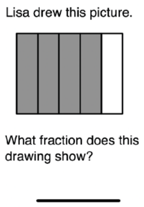
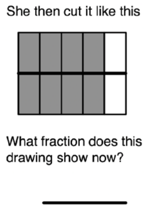
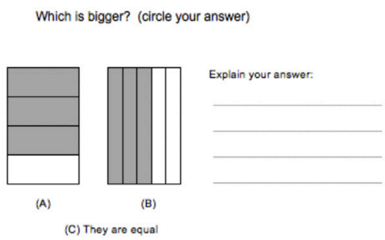
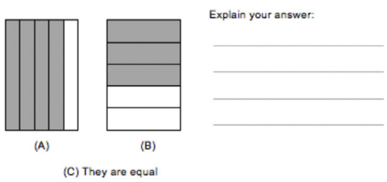
Appendix A

Questions and scoring criteria for the Screener

#	Question	Correctness	Unconventional Points	Not Unconventional
1	Draw a picture of $\frac{1}{2}$	1 point for a canonical drawing or representation of ½ (i.e., area model, number line, decimal, percent, or semicircle)	1 unconventional point for a drawing of a shape partitioned into two parts but without shading or labeling of either part.	A drawing of a shape partitioned into two with one of the parts labeled "½"
2	Draw another way to show $\frac{1}{2}$	(same as above)	(same as above)	(same as above)
3	Draw a picture of $\frac{3}{5}$	1 point for a canonical representation of 3/5 (e.g., area model, number line, or discrete set).	1 unconventional point for an area model or discrete set drawing of 2/5 (i.e., 3 out of 5 parts unshaded).	Partitioning issues, because students have difficulty accurately partitioning into fifths.
4	Draw a picture of $1\frac{5}{8}$	1 point for a canonical representation of 1 5/8 (e.g., area model, number line, or discrete set).	1 unconventional point for a representation where the whole is not shaded or labeled.	A drawing where the wholes are different sizes.
5	Circle all the pictures that you think show $\frac{1}{2}$? (correct answers circled below)  (adapted from Ni, 2001)	1 point for each correctly circled canonical representation of 1/2 (see circled answers) -1 point for each incorrect answer.	1 unconventional point for circling the halved circle with no shading.	
6	Which is more $\frac{1}{6}$ or $\frac{1}{8}$? (you can draw pictures to help you) Explain your answer:	1 point for correct answer (1/6) (explanations are used to disambiguate student answer, not required)	1 unconventional point for incorrect answer (1/8) with an explanation and/or drawing that focuses on the unshaded amount (e.g., more left in the 1/8 drawing).	A written answer that states that 1/8 is bigger than 1/6 because 8 is bigger than 6.
7	Which is more $\frac{2}{8}$ or $\frac{5}{8}$? Explain your answer:	1 point for correct answer (5/8) (explanations are used to disambiguate student answer, not required)	1 unconventional point for an incorrect answer (2/8) with an explanation and/or drawing that focuses on the fractional complement (e.g., 6/8 unshaded, 3/8 unshaded).	Answer of 2/8 with no explanation or drawing.

January 2022, Volume 14, Issue 3, 243-267

8		1 point for correct answer (4/5).	1 unconventional point for answers where the numerator is the number of parts not shaded (e.g., 1/5 or 1/4).	Answers where student has miscounted the number of pieces, (e.g., 5/6)
9		1 point for correct answer (8/10 or 4/5).	1 unconventional point for answers where the numerator is the number of parts not shaded (e.g., 2/10, 2/8, 1/5 or 1/4).	Answers where student has miscounted the number of pieces, (e.g., 10/12).
10	 (adapted from Armstrong & Larson, 1995)	1 point for correct answer (A) (explanations are used to disambiguate student answer, not required)	1 unconventional point for selecting B with an explanation focusing on the number "left" or unshaded amount.	An incorrect answer (B or C) with either no explanation or an explanation that suggests miscounting, (e.g., "C because 3/5=3/5")
11	 (adapted from Armstrong & Larson, 1995)	1 point for correct answer (A) (explanations are used to disambiguate student answer, not required)	1 unconventional point for selecting B with an explanation focusing on the number "left" or unshaded amount.	An incorrect answer (B or C) with either no explanation or an explanation that suggests miscounting, (e.g., "C because 3/5=3/5")
12	Solve the problem $\frac{1}{2}+\frac{1}{4}=$ using pictures.	1 point for correct answer (3/4). Student not required to draw pictures.	1 unconventional point for (a) answers that include a drawing of ½ without shading or (b) an answer of 2/4 (unshaded/shaded) with canonical area models of ½ and ¼.	An incorrect answer of 1/6 or 2/6 are not considered unconventional by themselves.
13	What fraction does this picture show? 	1 point for correct answer (e.g., 8/10 or 4/5).	1 unconventional point for an answers that determine the numerator based on the missing pieces (e.g., 2/10, 2/8, 1/5, 1/10, 1/8).	An incorrect answer in which the student has miscounted (e.g., 7/10 or 9/10).
Global coding: Any time the student interpreted a representation of as the fractional complement (e.g., interpreting 2/3 as 1/3) the student got an unconventional point for that problem.				

Appendix B

Background Interview Questions

Academic Background:

- What is your favorite subject in school?
- What do you like about it?
- What is your least favorite subject?
- What don't you like about it?
- What do you think about math? What do you like about it? What don't you like about it?

Nature of the student's difficulty:

- What are you working on in math right now?
- Can you give me an example?
- What about learning and/or doing math was hard for you? Can you give an example?
- What about learning and/or doing math was easy for you? Can you give an example?

Effort:

- Do you get a lot of homework in math?
- When do you do your homework?
- Do you tend to do all your homework and turn it in?

Resources Questions:

- If you get stuck on a problem, what do you do?
- Who do you ask for help, if you need it?

Language Fluency:

- What language do you tend to speak at home?